Laney-Math Lab

"The roots of education are bitter, but the fruit is sweet."[1]

DREAM.FLOURISH.SUCCEED.

1 Aristotle

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WELCOME TO LANEY-MATH LAB

Laney College educates, supports, and inspires students to excel in an inclusive and diverse learning environment rooted in social justice.

Website: https://laney.edu/mathematics/math-lab/

Location: Laney G-201

Email: Laney-Mathlab@peralta.edu

Tel: (510) 464-3448

Hours:

- Monday –Thursday 9:00 am to 7:00 pm
- Friday: TBA

About:

The Math Lab was created as a mathematics self-paced course facility. Now the Math Lab also operates as the department "information center" and offers drop-in tutoring for all math classes/problems. The Math Lab offers the following resources for students:

- Student tutors at all times, and one instructor on duty during selected hours, to assist students on a drop-in basis
- Table space and 20 computers for use (no printing)
- Textbooks available for checkout and use within the lab (photo ID is required, and we do not have a photocopier)
- Calculators available for checkout (photo ID required), expected to be returned within the same day.

Instructor contact information and mailboxes (in addition to the tower mailboxes)

MATH LAB ONLINE TUTORING

Laney Math Lab has been offering in-person tutoring to our diverse students to answer all their math problems. We are glad to offer online tutoring. If you can't make it to the Math Lab (Laney G201), you can now either call (510) 464-3448 or send us an email at laney-mathlab@peralta.edu to schedule an online tutoring session with one of our tutors. The session will take place on the Zoom app, so please make sure you follow these instructions:

- Create a free account on **https://zoom.us/**

- Download and install the online conference software from the website to your computer or smartphone

- We encourage you to play with all the functionality prior to a meeting so that you'll be familiar with the software to and make sure that your camera, speaker, and microphone work

- From your app, click on "Join a Meeting" and enter the Meting ID 468-550-8497 at your scheduled meeting time.

See you there!

ONLINE RESOURCES

In addition to getting tutoring help, there are a more resources available online.

- **PROFESSORS …**
- **www.laney.edu/mathematics**

The Laney Math department web page includes links to Laney professor web pages as well as plenty of other resources.

❑ ONLINE CALCULATORS …

- **www.desmos.com/calculator**

Desmos is a useful tool--an online graphing calculator that can also be downloaded as an app on mobile devices. The Desmos website features scientific calculators, and examples of graphic art created using its graphing calculators.

- **www.symbolab.com**

An "answer engine" like WolframAlpha (see below), but SymboLab's specialty is in providing step-by-step solutions for whatever problems a user enters into it.

- **www.geogebra.org**

A graphing calculator like Desmos, but more geared toward Trigonometry, with the ability to create and rotate shapes of all kinds.

- **www.wolframalpha.com**

According to Wikipedia, WolframAlpha isn't a calculator but rather a "computational knowledge engine." The home page prompts, "Enter what you want to calculate or know about," and the program offers answers based on its own database.

- **www.monroecc.edu/faculty/paulseeburger/calcnsf/CalcPlot3D/**

CalcPlot3D provides color 3D images which allow students to visualize the problems faced in multivariable calculus. A must for Calculus 3 students.

❑ ONLINE TUTORS …

- **www.khanacademy.org**

An online educational website that now includes instructional materials in 36 languages created by a staff of over 150 employees, Khan Academy (KA) offers help to students in all subjects. The math portion of the KA website includes videos and tutorials on Basic Math, Calculus, Statistics, Linear Algebra, and everything in between.

- **www.patrickjmt.com**

The creation of Patrick Jones, Patrick JMT is a website with HUNDREDS of Algebra and Calculus videos, as well as dozens of videos on several other math subjects.

- **www.mathisfun.com**

This website not only offers tutorials in Basic Math, Algebra, Geometry, and Pre-Calculus—it offers practice quizzes after practically every module! The Geometry resources are especially useful, and the on-site search engine makes things easy to find. Look for the Interactive Unit Circle or Geometric Constructions to enhance your understanding.

- **www.coolmath.com**

Includes links to a number of games, some math-related, for all ages, but also includes a lot of good explainers for Pre-Algebra, Algebra, and Pre-Calculus.

- **www.youtube.com**

A number of math tutors and/or tutorial services post the majority of their work on YouTube. Some channels worth checking out are:

  - **Professor Leonard**

Brandon Leonard, a professor of math at Merced College in California, has uploaded dozens of videos to his YouTube channel—his user name is professorleonard57. Leonard slows things down and teaches difficult subjects thoroughly, from Algebra on to Calculus I, II, and III.

  - **Gilbert Strang (MIT OpenCourseWare)**

Strang is an accomplished American mathematician who has been a professor at MIT for over 50 years. He teaches Linear Algebra, among other subjects, and his lectures are freely available online at the MIT OpenCourseWare channel.

  - **More from MIT OpenCourseWare**

- 18.01 Single Variable Calculus
- 18.02 Multivariable Calculus
- 18.03 Differential Equations
- 18.06 Linear Algebra (Gilbert Strang—see above)

- **Nancy Pi**

A series of videos mostly focusing on Algebra, PreCalculus, and some Calculus thrown in.

- **Blackpenredpen**

Another YouTube creator but this time with a focus on Calculus. The content creator at this YouTube channel, Steve, has fun doing math endurance videos like the one in which he did 100 integral problems in one six-hour sitting.

- **Video tutorial for using Statistics Calculators**

Example search terms such as "TI-84 regression tutorial," "TI-84 graphing tutorial," and "Excel regression tutorial" lead to a number of helpful videos.

- **3Blue1Brown**

This channel was created by 2015 Stanford math graduate Grant Sanderson, who briefly worked for Khan Academy before starting this channel—which focuses on explaining math concepts using animations. The site tilts toward upper level math, with one series of 12 videos on the essence of calculus, and another series of 15 videos on the essence of linear algebra.

- **Mathologer**

Math videos created by Australian math professor Burkhard Polster, covering a range of mathematical subjects.

- **www.mathtv.com**

Created by xyztextbooks, mathtv includes over 10,000 free math videos covering topics from basic math through Calculus. MathTV also has a YouTube channel, featuring videos by Pat McKeague, owner of both MathTV and XYZTextbooks.

- **www.myopenmath.com**

A free website with self-study courses in Pre-Algebra, Algebra, Precalculus, Trigonometry, and more.

❑ MISCELLANEOUS …

- **www.vihart.com**

The home page of self-described "mathe-musician" Vi Hart Includes a "giant list of videos" including mathematical musings about paper-folding, Fibonacci, pi, and the history of the Pythagorean Theorem.

- **www.studygs.net**

Study Guides and Strategies is the name of this web site, which includes a number of "study skills" applets but also includes a number of math modules mostly focused on Algebra skills.

- **www.numberphile.com**

Get ready to be amazed by the interviews on this website—most subjects are prestigious mathematicians sharing their work on the frontier of mathematical thought.

- **http://www.laney.edu/mathematics/amatyc-competition/**

For those interested in math competitions, Laney participates in the annual American Mathematical Association of Two-Year Colleges mathematics contest, also sponsored by the Student Mathematics League. The link above also offers resources for coaching to prepare for the contest.

❑ FOR TEACHERS …

- **www.mathed.page**

Created by math teacher, author, and speaker Henry Picciotto, this website includes tons of curricular materials for students of all ages, as well as Picciotto's thoughts on math instruction.

- **www.teach-kids-math-by-model-method.com/index.html**

Conquer any fear of word problems by using the "Model Method" known in many quarters as "Singapore Math." Students using the model method draw boxes to symbolize whole-part relationships. This website uses a number of examples and illustrations to introduce this approach—a great way to brush up on basic skills!

❑ IF YOU HAVE EXTRA CASH …

- **www.chegg.com**

An education student hub, Chegg offers tutors, reduced-price books, and homework help in a number of subjects, for a fee.

FORMULAS AND WORKSHEETS

Statistics Cheat Sheet

Population

The entire group one desires information about

Sample

A subset of the population taken because the entire population is usually too large to analyze
Its characteristics are taken to be representative of the population

Mean

Also called the arithmetic mean or average
The sum of all the values in the sample divided by the number of values in the sample/population
μ is the mean of the population; $\bar{x}$ is the mean of the sample

Median

The value separating the higher half of a sample/population from the lower half
Found by arranging all the values from lowest to highest and taking the middle one (or the mean of the middle two if there are an even number of values)

Variance

Measures dispersion around the mean
Determined by averaging the squared differences of all the values from the mean

Variance of a population is σ^2

$$\sigma^2 = \frac{\sum (x-\mu)^2}{n}$$

Can be calculated by subtracting the square of the mean from the average of the squared scores:

$$\sigma^2 = \frac{\sum x^2}{n} - \mu^2$$

Variance of a sample is s^2; note the n-1

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

Can be calculated by:

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

Standard Deviation

Square root of the variance
Also measures dispersion around the mean but in the same units as the values (instead of square units with variance)
σ is the standard deviation of the population and s is the standard deviation of the sample

Standard Error

An estimate of the standard deviation of the sampling distribution—the set of all samples of size n that can be taken from a population
Reflects the extent to which a statistic changes from sample to sample

For a mean, $\frac{s}{\sqrt{n}}$

For the difference between two means,

Assuming equal variances $\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$; unequal variances $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$

T-test

One-Sample

Tests whether the mean of a normally distributed population is different from a specified value

Null Hypothesis (H_0): states that the population mean is equal to some value (μ_0)
Alternative Hypothesis (H_a): states that the mean does not equal/is greater than/is less than μ_0
t-statistic: standardizes the difference between $\bar{x}$ and μ_0

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Degrees of freedom (df) = *n-1*

Read the table of t-distribution critical values for the p-value (probability that the sample mean was obtained by chance given μ_0 is the population mean) using the calculated t-statistic and degrees of freedom.

H_a: $\mu > \mu_0$ → the t-statistic is likely positive; read table as given

H_a: $\mu < \mu_0$ → the t-statistic is likely negative; the t-distribution is symmetrical so read the probability as if the t-statistic were positive

Note: if the t-statistic is of the 'wrong' sign, the p-value is 1 minus the *p* given in the chart

H_a: $\mu \neq \mu_0$ → read the p-value as if the t-statistic were positive and double it (to consider both less than and greater than)

If the p-value is less than the predetermined value for significance (called α and is usually 0.05), reject the null hypothesis and accept the alternative hypothesis.

Example:
You are experiencing hair loss and skin discoloration and think it might be because of selenium toxicity. You decide to measure the selenium levels in your tap water once a day for one week. Your results are given below. The EPA maximum contaminant level for safe drinking water is 0.05 mg/L. Does the selenium level in your tap water exceed the legal limit (assume α=0.05)?

Day	Selenium mg/L
1	0.051
2	0.0505
3	0.049
4	0.0516
5	0.052
6	0.0508
7	0.0506

H_0: μ=0.05; H_a: μ>0.05
Calculate the mean and standard deviation of your sample:

$$\bar{x} = 0.0508$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1} = \frac{(0.051 - 0.0508)^2 + (0.0505 - 0.0508)^2 + etc...}{6} = 9.15 \times 10^{-7}$$

$$s = \sqrt{s^2} = 9.56 \times 10^{-4}$$

The t-statistic is: $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{0.0508 - 0.05}{\frac{9.56 \times 10^{-4}}{\sqrt{7}}} = 2.17$ and the degrees of freedom are *n-1* = 7-1 = 6

Looking at the t-distribution of critical values table, 2.17 with 6 degrees of freedom is between *p*=0.05 and *p*=0.025. This means that the p-value is less than 0.05, so you can reject H_0 and conclude that the selenium level in your tap water exceeds the legal limit.

T-test

Two-Sample

Tests whether the means of two populations are significantly different from one another

Paired

Each value of one group corresponds directly to a value in the other group; ie: before and after values after drug treatment for each individual patient

Subtract the two values for each individual to get one set of values (the differences) and use μ_0 = 0 to perform a one-sample t-test

Unpaired

The two populations are independent

H_0: states that the means of the two populations are equal ($\mu_1 = \mu_2$)

H_a: states that the means of the two populations are unequal or one is greater than the other ($\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$, $\mu_1 < \mu_2$)

t-statistic:

assuming equal variances: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ assuming unequal variances: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)}}$

degrees of freedom = $(n_1-1)+(n_2-1)$

Read the table of t-distribution critical values for the p-value using the calculated t-statistic and degrees of freedom. Remember to keep the sign of the t-statistic clear (order of subtracting the sample means) and to double the p-value for an H_a of $\mu_1 \neq \mu_2$.

Example:

Consider the lifespan of 18 rats. 12 were fed a restricted calorie diet and lived an average of 700 days (standard deviation=21 days). The other 6 had unrestricted access to food and lived an average of 668 days (standard deviation=30 days). Does a restricted calorie diet increase the lifespan of rats (assume α=0.05)?

μ_1=700, s_1=21, n_1=12; μ_2=668, s_2=30, n_2=6

H_0: $\mu_1=\mu_2$

H_a: $\mu_1>\mu_2$ (because we are only asking if a restricted calorie diet increases lifespan)

We cannot assume that the variances of the two populations are equal because the different diets could also affect the variability in lifespan.

The t-statistic is: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)}} = \dfrac{700-668}{\sqrt{\dfrac{21^2}{12} + \dfrac{30^2}{6}}} = 2.342$

Degrees of freedom = $(n_1-1)+(n_2-1)$ = (12-1)+(6-1)=16

From the t-distribution table, the p-value falls between 0.01 and 0.02, so we do reject H_0. The restricted calorie diet does increase the lifespan of rats.

Chi-Square Test

For Goodness of Fit

Checks whether or not an observed pattern of data fits some given distribution

H_0: the observed pattern fits the given distribution

H_a: the observed pattern does not fit the given distribution

The chi-square statistic is: $\chi^2 = \sum \dfrac{(O-E)^2}{E}$ (*O* is the observed value and *E* is the expected value)

Degrees of freedom = number of categories in the distribution – 1

Get the p-value from the table of χ^2 critical values using the calculated χ^2 and df values. If the p-value is less than α, the observed data does not fit the expected distribution. If $p>\alpha$, the data likely fits the expected distribution

Example 1:

You breed puffskeins and would like to determine the pattern of inheritance for coat color and purring ability. Puffskeins come in either pink or purple and can either purr or hiss. You breed a purebred, pink purring male with a purebred, purple hissing female. All individuals of the F_1 generation are pink and purring. The F_2 offspring are shown below. Do the alleles for coat color and purring ability assort independently (assume α=0.05)?

Pink and Purring	Pink and Hissing	Purple and Purring	Purple and Hissing
143	60	55	18

Independent assortment means a phenotypic ratio of 9:3:3:1, so:

H_0: the observed distribution of F_2 offspring fits a 9:3:3:1 distribution

H_a: the observed distribution of F_2 offspring does not fit a 9:3:3:1 distribution

The expected values are:

Pink and Purring	Pink and Hissing	Purple and Purring	Purple and Hissing
155.25	51.75	51.75	17.25

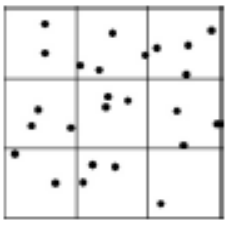
$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(143-155.25)^2}{155.25} + \frac{(60-51.75)^2}{51.75} + \frac{(55-51.75)^2}{51.75} + \frac{(18-17.25)^2}{17.25} = 2.519$$

df=4-1=3

From the table of χ^2 critical values, the p-value is greater than 0.25, so the alleles for coat color and purring ability do assort independently in puffskeins.

Example 2:

You are studying the pattern of dispersion of king penguins and the diagram on the right represents an area you sampled. Each dot is a penguin. Do the penguins display a uniform distribution (assume α=0.05)?



H_0: there is a uniform distribution of penguins

H_a: there is not a uniform distribution of penguins

There are a total of 25 penguins, so if there is a uniform distribution, there should be 2.778 penguins per square. There actual observed values are 2, 4, 4, 3, 3, 3, 2, 3, 1, so the χ^2 statistic is:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(1-2.778)^2}{2.778} + 2\left(\frac{(2-2.778)^2}{2.778}\right) + 4\left(\frac{(3-2.778)^2}{2.778}\right) + 2\left(\frac{(4-2.778)^2}{2.778}\right) = 2.72$$

df=9-1=8

From the table of χ^2 critical values, the p-value is greater than 0.25, so we do not reject H_0. The penguins do display a uniform distribution.

Chi-Square Test

For Independence

Checks whether two categorical variables are related or not (independence)

H_0: the two variables are independent

H_a: the two variables are not independent

Does not make any assumptions about an expected distribution

The observed values ($\#_1$, $\#_2$, $\#_3$, and $\#_4$) are usually presented as a table. Each row is a category of variable 1 and each column is a category of variable 2.

		Variable 1		Totals
		Category *x*	Category *y*	
Variable 2	Category *a*	$\#_1$	$\#_2$	$\#_1+\#_2$
	Category *b*	$\#_3$	$\#_4$	$\#_3+\#_4$
Totals		$\#_1+\#_3$	$\#_2+\#_4$	$\#_1+\#_2+\#_3+\#_4$

The proportion of category *x* of variable 1 is the number of individuals in category *x* divided by the total number of individuals $\left(\frac{\#_1+\#_3}{\#_1+\#_2+\#_3+\#_4}\right)$. Assuming independence, the expected number of individuals that fall within category *a* of variable 2 is the proportion of category *x* multiplied by the number of individuals in category *a* $\left(\frac{\#_1+\#_3}{\#_1+\#_2+\#_3+\#_4}\right)(\#_1+\#_2)$. Thus, the expected value is:

$$E = \frac{(\#_1+\#_3)(\#_1+\#_2)}{\#_1+\#_2+\#_3+\#_4} = \frac{(row\ total)(column\ total)}{grand\ total}$$

Degrees of freedom = (*r*-1)(*c*-1) where *r* is the number of rows and *c* is the number of columns

The chi-square statistic is still $\chi^2 = \sum \frac{(O-E)^2}{E}$

Read the p-values from the table of χ^2 critical values.

Example:

Given the data below, is there a relationship between fitness level and smoking habits (assume α=0.05)?

	Fitness Level				
	Low	Medium-Low	Medium-High	High	
Never smoked	113	113	110	159	495
Former smokers	119	135	172	190	616
1 to 9 cigarettes daily	77	91	86	65	319
≥ 10 cigarettes daily	181	152	124	73	530
	490	491	492	487	1960

H_0: fitness level and smoking habits are independent
H_a: fitness level and smoking habits are not independent
First, we calculate the expected counts. For the first cell, the expected count is:

$$E = \frac{(row\ total)(column\ total)}{grand\ total} = \frac{(495)(490)}{1960} = 123.75$$

	Fitness Level			
	Low	Medium-Low	Medium-High	High
Never smoked	123.75	124	124.26	122.99
Former smokers	154	154.31	154.63	153.06
1 to 9 cigarettes daily	79.75	79.91	80.08	79.26
≥ 10 cigarettes daily	132.5	132.77	133.04	131.69

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(113-123.75)^2}{123.75} + \frac{(113-124)^2}{124} + \frac{(110-124.26)^2}{124.26} + etc\ldots = 91.73$$

df=(r-1)(c-1)=(4-1)(4-1)=9

From the table of χ^2 critical values, the p-value is less than 0.001, so we reject H_0 and conclude that there is a relationship between fitness level and smoking habits.

Type I error

The probability of rejecting a true null hypothesis
Equals α

Type II error

The probability of failing to reject a false null hypothesis

Probability

Joint Probability

The probability of events A and B occurring
$P(A \text{ and } B) = P(A) \times P(B)$ when events A and B are independent

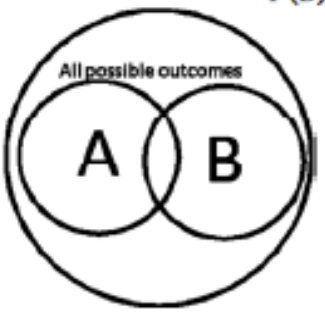
Union of Events

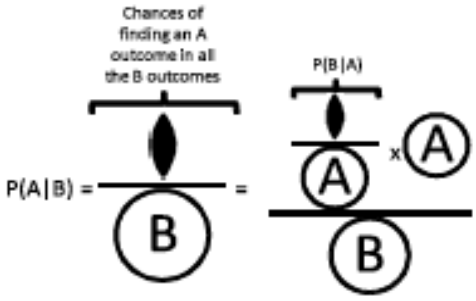
The probability of either event A or event B occurring
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Conditional Probability

The probability of event A occurring given that event B has occurred

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{or} \quad P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$





Example 1:
Assume that eye color is an autosomally inherited trait controlled by one gene with two alleles. Brown is dominant to blue. A brown-eyed man with genotype Bb and a blue-eyed woman have three children. The first has blue eyes. What is the probability that all three children have blue eyes?

Without considering the first child, the probability that the couple has three children with blue eyes is
$0.5 \times 0.5 \times 0.5 = 0.125 = P(A \text{ and } B) = P(2 \text{ children} = \text{bb and 1st child bb})$
With his parents, the probability that the 1st child is bb is: $P(B) = P(1\text{st child} = \text{bb}) = 0.5$
Therefore, $P(2 \text{ children} = \text{bb} \mid \text{1st child bb}) = P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.125}{0.5} 0.25$

Example 2:
Based on an analysis of her pedigree, it is determined that a woman has a 70% chance of being Zz and a 30% chance of being ZZ for a sex-linked trait, where Z is dominant to z. If she now has a son with the Z phenotype, what is the probability of her being Zz?

We're looking for: P(W=Zz|S=Z)
But it's hard to find P(W=Zz and S=Z) because the two events are not independent. Instead, let us use:

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

$P(S = Z \mid W = Zz) = 0.5$ (50% chance of passing on the Z allele)
$P(W = Zz) = 0.7$ (given)
$P(S = Z) = (0.7 \times 0.5) + (0.3 \times 1) = 0.65$ (son can be Z from the woman being either Zz or ZZ)

$$P(W = Zz \mid S = Z) = \frac{0.5 \times 0.7}{0.65} = 0.538$$

Multiple Experiments

Binomial distribution

For when you are not concerned about the order of the events, only that they occur

$$P(X = m) = \frac{n! \times p^m \times (1-p)^{(n-m)}}{m! \times (n-m)!}$$

for m outcomes of event X in n total trials with p=probability of X occurring once

Example:
What is the probability that a couple has one boy out of five children?

$$P(1 \text{ boy of 5 children}) = \frac{5! \times 0.5^1 \times 0.5^4}{1! \times (4)!} = 0.15625$$

Poisson distribution

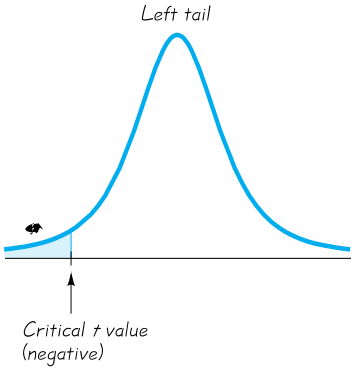
The binomial distribution works for a small number of trials but as n gets too large, the factorials become unwieldy.
The Poisson distribution is an estimate of the binomial distribution for large n.

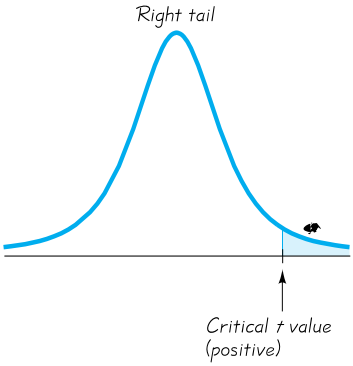
$$P(X = m) = \frac{e^{-np} \times (n \times p)^m}{m!}$$

Note: np is also known as the number of expected outcomes for event X

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319
24	**2.797**	2.492	**2.064**	1.711	1.318
25	**2.787**	2.485	**2.060**	1.708	1.316
26	**2.779**	2.479	**2.056**	1.706	1.315
27	**2.771**	2.473	**2.052**	1.703	1.314
28	**2.763**	2.467	**2.048**	1.701	1.313
29	**2.756**	2.462	**2.045**	1.699	1.311
30	**2.750**	2.457	**2.042**	1.697	1.310
31	**2.744**	2.453	**2.040**	1.696	1.309
32	**2.738**	2.449	**2.037**	1.694	1.309
34	**2.728**	2.441	**2.032**	1.691	1.307
36	**2.719**	2.434	**2.028**	1.688	1.306
38	**2.712**	2.429	**2.024**	1.686	1.304
40	**2.704**	2.423	**2.021**	1.684	1.303
45	**2.690**	2.412	**2.014**	1.679	1.301
50	**2.678**	2.403	**2.009**	1.676	1.299
55	**2.668**	2.396	**2.004**	1.673	1.297
60	**2.660**	2.390	**2.000**	1.671	1.296
65	**2.654**	2.385	**1.997**	1.669	1.295
70	**2.648**	2.381	**1.994**	1.667	1.294
75	**2.643**	2.377	**1.992**	1.665	1.293
80	**2.639**	2.374	**1.990**	1.664	1.292
90	**2.632**	2.368	**1.987**	1.662	1.291
100	**2.626**	2.364	**1.984**	1.660	1.290
200	**2.601**	2.345	**1.972**	1.653	1.286
300	**2.592**	2.339	**1.968**	1.650	1.284
400	**2.588**	2.336	**1.966**	1.649	1.284
500	**2.586**	2.334	**1.965**	1.648	1.283
750	**2.582**	2.331	**1.963**	1.647	1.283
1000	**2.581**	2.330	**1.962**	1.646	1.282
2000	**2.578**	2.328	**1.961**	1.646	1.282
Large	**2.576**	2.326	**1.960**	1.645	1.282





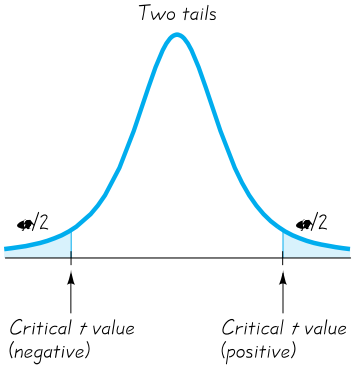


Table entry for p is the critical value $(\chi^2)^*$ with probability p lying to its right.

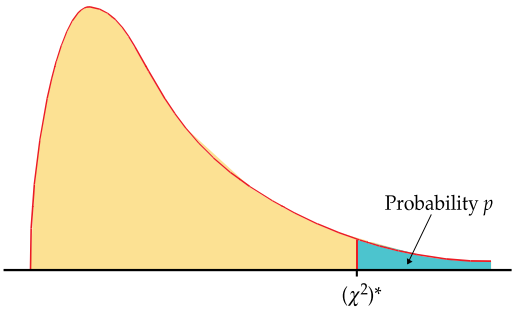


TABLE F

χ^2 distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

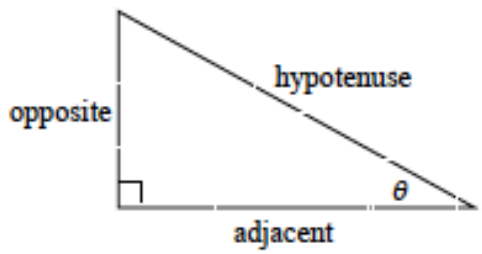
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



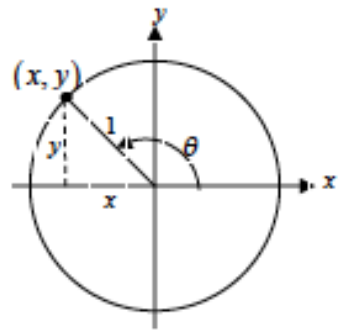
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ $\quad$ $\sin\theta = \dfrac{1}{\csc\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\quad$ $\cos\theta = \dfrac{1}{\sec\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\quad$ $\tan\theta = \dfrac{1}{\cot\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even/Odd Formulas

$\sin(-\theta) = -\sin\theta$ $\quad$ $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Periodic Formulas

If n is an integer.

$\sin(\theta + 2\pi n) = \sin\theta$ $\quad$ $\csc(\theta + 2\pi n) = \csc\theta$

$\cos(\theta + 2\pi n) = \cos\theta$ $\quad$ $\sec(\theta + 2\pi n) = \sec\theta$

$\tan(\theta + \pi n) = \tan\theta$ $\quad$ $\cot(\theta + \pi n) = \cot\theta$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas (alternate form)

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\quad$ $\sin^2\theta = \dfrac{1}{2}(1-\cos(2\theta))$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\quad$ $\cos^2\theta = \dfrac{1}{2}(1+\cos(2\theta))$

$\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$ $\quad$ $\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Product to Sum Formulas

$\sin\alpha\sin\beta = \dfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

Sum to Product Formulas

$\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\dfrac{\alpha+\beta}{2}\right)\sin\left(\dfrac{\alpha-\beta}{2}\right)$

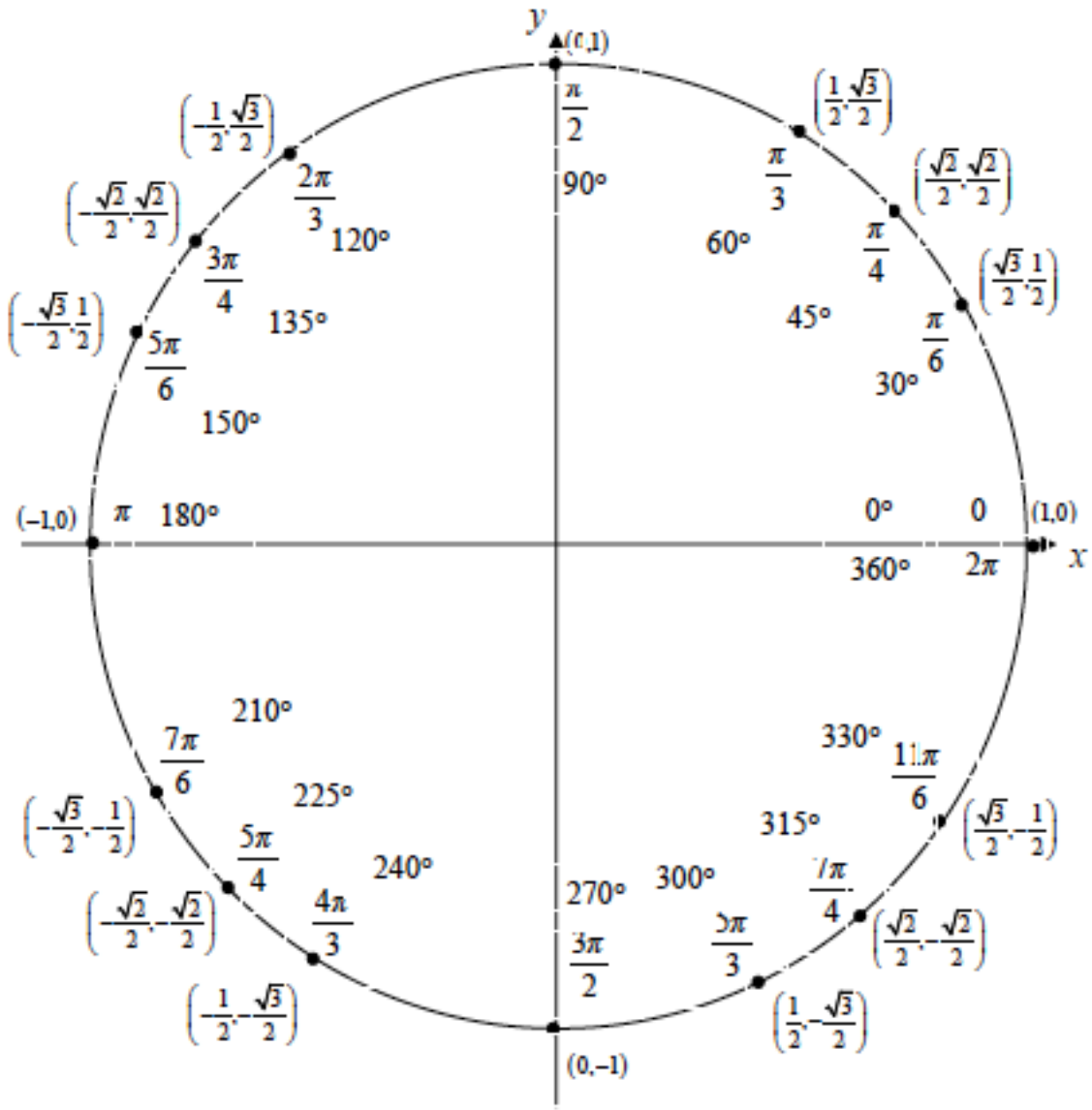
Cofunction Formulas

$\sin\left(\dfrac{\pi}{2}-\theta\right) = \cos\theta$ $\quad$ $\cos\left(\dfrac{\pi}{2}-\theta\right) = \sin\theta$

$\csc\left(\dfrac{\pi}{2}-\theta\right) = \sec\theta$ $\quad$ $\sec\left(\dfrac{\pi}{2}-\theta\right) = \csc\theta$

$\tan\left(\dfrac{\pi}{2}-\theta\right) = \cot\theta$ $\quad$ $\cot\left(\dfrac{\pi}{2}-\theta\right) = \tan\theta$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos(\cos^{-1}(x)) = x \qquad \cos^{-1}(\cos(\theta)) = \theta$

$\sin(\sin^{-1}(x)) = x \qquad \sin^{-1}(\sin(\theta)) = \theta$

$\tan(\tan^{-1}(x)) = x \qquad \tan^{-1}(\tan(\theta)) = \theta$

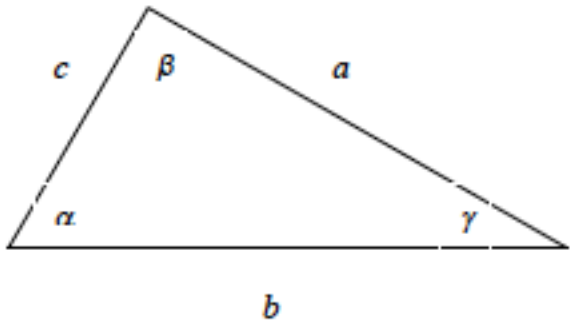
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

Calculus Cheat Sheet

Limits

Definitions

Precise Definition : We say $\lim_{x\to a} f(x) = L$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that whenever $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$.

"Working" Definition : We say $\lim_{x\to a} f(x) = L$ if we can make $f(x)$ as close to L as we want by taking x sufficiently close to a (on either side of a) without letting $x = a$.

Right hand limit : $\lim_{x\to a^+} f(x) = L$. This has the same definition as the limit except it requires $x > a$.

Left hand limit : $\lim_{x\to a^-} f(x) = L$. This has the same definition as the limit except it requires $x < a$.

Limit at Infinity : We say $\lim_{x\to\infty} f(x) = L$ if we can make $f(x)$ as close to L as we want by taking x large enough and positive.

There is a similar definition for $\lim_{x\to-\infty} f(x) = L$ except we require x large and negative.

Infinite Limit : We say $\lim_{x\to a} f(x) = \infty$ if we can make $f(x)$ arbitrarily large (and positive) by taking x sufficiently close to a (on either side of a) without letting $x = a$.

There is a similar definition for $\lim_{x\to a} f(x) = -\infty$ except we make $f(x)$ arbitrarily large and negative.

Relationship between the limit and one-sided limits

$$\lim_{x\to a} f(x) = L \;\Rightarrow\; \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L \qquad \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L \;\Rightarrow\; \lim_{x\to a} f(x) = L$$

$$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x) \;\Rightarrow\; \lim_{x\to a} f(x) \text{ Does Not Exist}$$

Properties

Assume $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and c is any number then,

1. $\lim_{x\to a}\left[cf(x)\right] = c\lim_{x\to a} f(x)$
2. $\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
3. $\lim_{x\to a}\left[f(x)g(x)\right] = \lim_{x\to a} f(x)\ \lim_{x\to a} g(x)$
4. $\lim_{x\to a}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided $\lim_{x\to a} g(x) \neq 0$
5. $\lim_{x\to a}\left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$
6. $\lim_{x\to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{\lim_{x\to a} f(x)}$

Basic Limit Evaluations at $\pm\infty$

Note : $\operatorname{sgn}(a) = 1$ if $a > 0$ and $\operatorname{sgn}(a) = -1$ if $a < 0$.

1. $\lim_{x\to\infty} \mathbf{e}^x = \infty$ & $\lim_{x\to-\infty} \mathbf{e}^x = 0$
2. $\lim_{x\to\infty} \ln(x) = \infty$ & $\lim_{x\to 0^+} \ln(x) = -\infty$
3. If $r > 0$ then $\lim_{x\to\infty} \dfrac{b}{x^r} = 0$
4. If $r > 0$ and x^r is real for negative x then $\lim_{x\to-\infty} \dfrac{b}{x^r} = 0$
5. n even : $\lim_{x\to\pm\infty} x^n = \infty$
6. n odd : $\lim_{x\to\infty} x^n = \infty$ & $\lim_{x\to-\infty} x^n = -\infty$
7. n even : $\lim_{x\to\pm\infty} a x^n + \cdots + b x + c = \operatorname{sgn}(a)\infty$
8. n odd : $\lim_{x\to\infty} a x^n + \cdots + b x + c = \operatorname{sgn}(a)\infty$
9. n odd : $\lim_{x\to-\infty} a x^n + \cdots + c x + d = -\operatorname{sgn}(a)\infty$

Visit http://tutorial.math.lamar.edu for a complete set of Calculus notes.

Calculus Cheat Sheet

Evaluation Techniques

Continuous Functions

If $f(x)$ is continuous at a then $\lim_{x\to a} f(x) = f(a)$

Continuous Functions and Composition

$f(x)$ is continuous at b and $\lim_{x\to a} g(x) = b$ then

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right) = f(b)$$

Factor and Cancel

$$\lim_{x\to 2}\frac{x^2+4x-12}{x^2-2x} = \lim_{x\to 2}\frac{(x-2)(x+6)}{x(x-2)}$$

$$= \lim_{x\to 2}\frac{x+6}{x} = \frac{8}{2} = 4$$

Rationalize Numerator/Denominator

$$\lim_{x\to 9}\frac{3-\sqrt{x}}{x^2-81} = \lim_{x\to 9}\frac{3-\sqrt{x}}{x^2-81}\,\frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$= \lim_{x\to 9}\frac{9-x}{\left(x^2-81\right)\left(3+\sqrt{x}\right)} = \lim_{x\to 9}\frac{-1}{(x+9)\left(3+\sqrt{x}\right)}$$

$$= \frac{-1}{(18)(6)} = -\frac{1}{108}$$

Combine Rational Expressions

$$\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{-h}{x(x+h)}\right) = \lim_{h\to 0}\frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

L'Hospital's Rule

If $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$ then,

$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ a is a number, ∞ or $-\infty$

Polynomials at Infinity

$p(x)$ and $q(x)$ are polynomials. To compute $\lim_{x\to\pm\infty}\frac{p(x)}{q(x)}$ factor largest power of x in $q(x)$ out of both $p(x)$ and $q(x)$ then compute limit.

$$\lim_{x\to-\infty}\frac{3x^2-4}{5x-2x^2} = \lim_{x\to-\infty}\frac{x^2\left(3-\frac{4}{x^2}\right)}{x^2\left(\frac{5}{x}-2\right)} = \lim_{x\to-\infty}\frac{3-\frac{4}{x^2}}{\frac{5}{x}-2} = -\frac{3}{2}$$

Piecewise Function

$\lim_{x\to -2} g(x)$ where $g(x) = \begin{cases} x^2+5 & \text{if } x<-2 \\ 1-3x & \text{if } x\ge -2 \end{cases}$

Compute two one sided limits,

$$\lim_{x\to -2^-} g(x) = \lim_{x\to -2^-} x^2+5 = 9$$

$$\lim_{x\to -2^+} g(x) = \lim_{x\to -2^+} 1-3x = 7$$

One sided limits are different so $\lim_{x\to -2} g(x)$ doesn't exist. If the two one sided limits had been equal then $\lim_{x\to -2} g(x)$ would have existed and had the same value.

Some Continuous Functions

Partial list of continuous functions and the values of x for which they are continuous.

1. Polynomials for all x.
2. Rational function, except for x's that give division by zero.
3. $\sqrt[n]{x}$ (n odd) for all x.
4. $\sqrt[n]{x}$ (n even) for all $x \ge 0$.
5. $\mathbf{e}^x$ for all x.
6. $\ln x$ for $x > 0$.
7. $\cos(x)$ and $\sin(x)$ for all x.
8. $\tan(x)$ and $\sec(x)$ provided $x \ne \cdots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots$
9. $\cot(x)$ and $\csc(x)$ provided $x \ne \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots$

Intermediate Value Theorem

Suppose that $f(x)$ is continuous on [a, b] and let M be any number between $f(a)$ and $f(b)$. Then there exists a number c such that $a < c < b$ and $f(c) = M$.

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Calculus Cheat Sheet

Derivatives

Definition and Notation

If $y = f(x)$ then the derivative is defined to be $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

If $y = f(x)$ then all of the following are equivalent notations for the derivative.

$$f'(x) = y' = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx}\left(f(x)\right) = Df(x)$$

If $y = f(x)$ all of the following are equivalent notations for derivative evaluated at $x = a$.

$$f'(a) = y'\Big|_{x=a} = \frac{df}{dx}\Big|_{x=a} = \frac{dy}{dx}\Big|_{x=a} = Df(a)$$

Interpretation of the Derivative

If $y = f(x)$ then,

1. $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$ and the equation of the tangent line at $x = a$ is given by $y = f(a) + f'(a)(x - a)$.
2. $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$.
3. If $f(x)$ is the position of an object at time x then $f'(a)$ is the velocity of the object at $x = a$.

Basic Properties and Formulas

If $f(x)$ and $g(x)$ are differentiable functions (the derivative exists), c and n are any real numbers,

1. $(c f)' = c f'(x)$
2. $(f \pm g)' = f'(x) \pm g'(x)$
3. $(f g)' = f' g + f g'$ **– Product Rule**
4. $\left(\frac{f}{g}\right)' = \frac{f' g - f g'}{g^2}$ **– Quotient Rule**
5. $\frac{d}{dx}(c) = 0$
6. $\frac{d}{dx}(x^n) = n x^{n-1}$ **– Power Rule**
7. $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\, g'(x)$
   This is the **Chain Rule**

Common Derivatives

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$

$\frac{d}{dx}(a^x) = a^x \ln(a)$

$\frac{d}{dx}(\mathbf{e}^x) = \mathbf{e}^x$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, \quad x > 0$

$\frac{d}{dx}(\ln|x|) = \frac{1}{x}, \quad x \neq 0$

$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln a}, \quad x > 0$

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Calculus Cheat Sheet

Chain Rule Variants

The chain rule applied to some specific functions.

1. $\frac{d}{dx}\left(\left[f(x)\right]^n\right) = n\left[f(x)\right]^{n-1} f'(x)$
2. $\frac{d}{dx}\left(\mathbf{e}^{f(x)}\right) = f'(x)\mathbf{e}^{f(x)}$
3. $\frac{d}{dx}\left(\ln\left[f(x)\right]\right) = \frac{f'(x)}{f(x)}$
4. $\frac{d}{dx}\left(\sin\left[f(x)\right]\right) = f'(x)\cos\left[f(x)\right]$
5. $\frac{d}{dx}\left(\cos\left[f(x)\right]\right) = -f'(x)\sin\left[f(x)\right]$
6. $\frac{d}{dx}\left(\tan\left[f(x)\right]\right) = f'(x)\sec^2\left[f(x)\right]$
7. $\frac{d}{dx}\left(\sec\left[f(x)\right]\right) = f'(x)\sec\left[f(x)\right]\tan\left[f(x)\right]$
8. $\frac{d}{dx}\left(\tan^{-1}\left[f(x)\right]\right) = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Higher Order Derivatives

The Second Derivative is denoted as

$f''(x) = f^{(2)}(x) = \frac{d^2 f}{dx^2}$ and is defined as

$f''(x) = \left(f'(x)\right)'$, *i.e.* the derivative of the first derivative, $f'(x)$.

The nth Derivative is denoted as

$f^{(n)}(x) = \frac{d^n f}{dx^n}$ and is defined as

$f^{(n)}(x) = \left(f^{(n-1)}(x)\right)'$, *i.e.* the derivative of the (n-1)st derivative, $f^{(n-1)}(x)$.

Implicit Differentiation

Find y' if $\mathbf{e}^{2x-9y} + x^3y^2 = \sin(y) + 11x$. Remember $y = y(x)$ here, so products/quotients of x and y will use the product/quotient rule and derivatives of y will use the chain rule. The "trick" is to differentiate as normal and every time you differentiate a y you tack on a y' (from the chain rule). After differentiating solve for y'.

$$\begin{aligned}
&\mathbf{e}^{2x-9y}(2-9y') + 3x^2y^2 + 2x^3y\,y' = \cos(y)\,y' + 11 \\
&2\mathbf{e}^{2x-9y} - 9y'\mathbf{e}^{2x-9y} + 3x^2y^2 + 2x^3y\,y' = \cos(y)\,y' + 11 \qquad \Rightarrow \qquad y' = \frac{11 - 2\mathbf{e}^{2x-9y} - 3x^2y^2}{2x^3y - 9\mathbf{e}^{2x-9y} - \cos(y)} \\
&\left(2x^3y - 9\mathbf{e}^{2x-9y} - \cos(y)\right)y' = 11 - 2\mathbf{e}^{2x-9y} - 3x^2y^2
\end{aligned}$$

Increasing/Decreasing – Concave Up/Concave Down

Critical Points

$x = c$ is a critical point of $f(x)$ provided either **1.** $f'(c) = 0$ or **2.** $f'(c)$ doesn't exist.

Increasing/Decreasing

1. If $f'(x) > 0$ for all x in an interval I then $f(x)$ is increasing on the interval I.
2. If $f'(x) < 0$ for all x in an interval I then $f(x)$ is decreasing on the interval I.
3. If $f'(x) = 0$ for all x in an interval I then $f(x)$ is constant on the interval I.

Concave Up/Concave Down

1. If $f''(x) > 0$ for all x in an interval I then $f(x)$ is concave up on the interval I.
2. If $f''(x) < 0$ for all x in an interval I then $f(x)$ is concave down on the interval I.

Inflection Points

$x = c$ is a inflection point of $f(x)$ if the concavity changes at $x = c$.

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Calculus Cheat Sheet

Extrema

Absolute Extrema

1. $x = c$ is an absolute maximum of $f(x)$ if $f(c) \geq f(x)$ for all x in the domain.
2. $x = c$ is an absolute minimum of $f(x)$ if $f(c) \leq f(x)$ for all x in the domain.

Fermat's Theorem

If $f(x)$ has a relative (or local) extrema at $x = c$, then $x = c$ is a critical point of $f(x)$.

Extreme Value Theorem

If $f(x)$ is continuous on the closed interval $[a,b]$ then there exist numbers c and d so that, **1.** $a \leq c, d \leq b$, **2.** $f(c)$ is the abs. max. in $[a,b]$, **3.** $f(d)$ is the abs. min. in $[a,b]$.

Finding Absolute Extrema

To find the absolute extrema of the continuous function $f(x)$ on the interval $[a,b]$ use the following process.

1. Find all critical points of $f(x)$ in $[a,b]$.
2. Evaluate $f(x)$ at all points found in Step 1.
3. Evaluate $f(a)$ and $f(b)$.
4. Identify the abs. max. (largest function value) and the abs. min.(smallest function value) from the evaluations in Steps 2 & 3.

Relative (local) Extrema

1. $x = c$ is a relative (or local) maximum of $f(x)$ if $f(c) \geq f(x)$ for all x near c.
2. $x = c$ is a relative (or local) minimum of $f(x)$ if $f(c) \leq f(x)$ for all x near c.

1st Derivative Test

If $x = c$ is a critical point of $f(x)$ then $x = c$ is

1. a rel. max. of $f(x)$ if $f'(x) > 0$ to the left of $x = c$ and $f'(x) < 0$ to the right of $x = c$.
2. a rel. min. of $f(x)$ if $f'(x) < 0$ to the left of $x = c$ and $f'(x) > 0$ to the right of $x = c$.
3. not a relative extrema of $f(x)$ if $f'(x)$ is the same sign on both sides of $x = c$.

2nd Derivative Test

If $x = c$ is a critical point of $f(x)$ such that $f'(c) = 0$ then $x = c$

1. is a relative maximum of $f(x)$ if $f''(c) < 0$.
2. is a relative minimum of $f(x)$ if $f''(c) > 0$.
3. may be a relative maximum, relative minimum, or neither if $f''(c) = 0$.

Finding Relative Extrema and/or Classify Critical Points

1. Find all critical points of $f(x)$.
2. Use the 1st derivative test or the 2nd derivative test on each critical point.

Mean Value Theorem

If $f(x)$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b) then there is a number $a < c < b$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Newton's Method

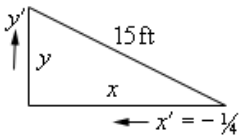
If x_n is the nth guess for the root/solution of $f(x) = 0$ then $(n+1)^{st}$ guess is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ provided $f'(x_n)$ exists.

Calculus Cheat Sheet

Related Rates

Sketch picture and identify known/unknown quantities. Write down equation relating quantities and differentiate with respect to t using implicit differentiation (*i.e.* add on a derivative every time you differentiate a function of t). Plug in known quantities and solve for the unknown quantity.

Ex. A 15 foot ladder is resting against a wall. The bottom is initially 10 ft away and is being pushed towards the wall at $\frac{1}{4}$ ft/sec. How fast is the top moving after 12 sec?



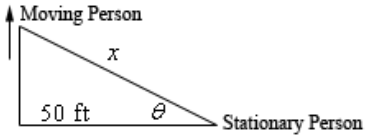
x' is negative because x is decreasing. Using Pythagorean Theorem and differentiating,

$$x^2 + y^2 = 15^2 \quad \Rightarrow \quad 2x\,x' + 2y\,y' = 0$$

After 12 sec we have $x = 10 - 12\left(\frac{1}{4}\right) = 7$ and so $y = \sqrt{15^2 - 7^2} = \sqrt{176}$. Plug in and solve for y'.

$$7\left(-\tfrac{1}{4}\right) + \sqrt{176}\,y' = 0 \quad \Rightarrow \quad y' = \frac{7}{4\sqrt{176}} \text{ ft/sec}$$

Ex. Two people are 50 ft apart when one starts walking north. The angle θ changes at 0.01 rad/min. At what rate is the distance between them changing when $\theta = 0.5$ rad?



We have $\theta' = 0.01$ rad/min. and want to find x'. We can use various trig fcns but easiest is,

$$\sec\theta = \frac{x}{50} \quad \Rightarrow \quad \sec\theta\tan\theta\,\theta' = \frac{x'}{50}$$

We know $\theta = 0.5$ so plug in θ' and solve.

$$\sec(0.5)\tan(0.5)(0.01) = \frac{x'}{50}$$

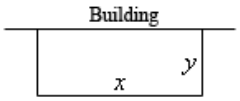
$$x' = 0.3112 \text{ ft/sec}$$

Remember to have calculator in radians!

Optimization

Sketch picture if needed, write down equation to be optimized and constraint. Solve constraint for one of the two variables and plug into first equation. Find critical points of equation in range of variables and verify that they are min/max as needed.

Ex. We're enclosing a rectangular field with 500 ft of fence material and one side of the field is a building. Determine dimensions that will maximize the enclosed area.



Maximize $A = xy$ subject to constraint of $x + 2y = 500$. Solve constraint for x and plug into area.

$$x = 500 - 2y \quad \Rightarrow \quad A = y(500 - 2y) = 500y - 2y^2$$

Differentiate and find critical point(s).

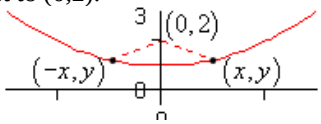
$$A' = 500 - 4y \quad \Rightarrow \quad y = 125$$

By 2nd deriv. test this is a rel. max. and so is the answer we're after. Finally, find x.

$$x = 500 - 2(125) = 250$$

The dimensions are then 250 x 125.

Ex. Determine point(s) on $y = x^2 + 1$ that are closest to (0,2).



Minimize $f = d^2 = (x - 0)^2 + (y - 2)^2$ and the constraint is $y = x^2 + 1$. Solve constraint for x^2 and plug into the function.

$$x^2 = y - 1 \quad \Rightarrow \quad f = x^2 + (y - 2)^2$$

$$= y - 1 + (y - 2)^2 = y^2 - 3y + 3$$

Differentiate and find critical point(s).

$$f' = 2y - 3 \quad \Rightarrow \quad y = \tfrac{3}{2}$$

By the 2nd derivative test this is a rel. min. and so all we need to do is find x value(s).

$$x^2 = \tfrac{3}{2} - 1 = \tfrac{1}{2} \quad \Rightarrow \quad x = \pm\tfrac{1}{\sqrt{2}}$$

The 2 points are then $\left(\frac{1}{\sqrt{2}}, \frac{3}{2}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{3}{2}\right)$.

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Calculus Cheat Sheet

Integrals

Definitions

Definite Integral: Suppose $f(x)$ is continuous on $[a,b]$. Divide $[a,b]$ into n subintervals of width Δx and choose x_i^* from each interval. Then $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$.

Anti-Derivative : An anti-derivative of $f(x)$ is a function, $F(x)$, such that $F'(x) = f(x)$.

Indefinite Integral : $\int f(x)\,dx = F(x) + c$ where $F(x)$ is an anti-derivative of $f(x)$.

Fundamental Theorem of Calculus

Part I : If $f(x)$ is continuous on $[a,b]$ then $g(x) = \int_a^x f(t)\,dt$ is also continuous on $[a,b]$ and $g'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Part II : $f(x)$ is continuous on $[a,b]$, $F(x)$ is an anti-derivative of $f(x)$ (*i.e.* $F(x) = \int f(x)\,dx$) then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Variants of Part I :

$$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = u'(x)\,f[u(x)]$$

$$\frac{d}{dx}\int_{v(x)}^{b} f(t)\,dt = -v'(x)\,f[v(x)]$$

$$\frac{d}{dx}\int_{v(x)}^{u(x)} f(t)\,dt = u'(x)\,f[u(x)] - v'(x)\,f[v(x)]$$

Properties

$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int cf(x)\,dx = c\int f(x)\,dx$, c is a constant

$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, c is a constant

$\int_a^a f(x)\,dx = 0$

$\int_a^b c\,dx = c(b-a)$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for any value of c.

If $f(x) \ge g(x)$ on $a \le x \le b$ then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

If $f(x) \ge 0$ on $a \le x \le b$ then $\int_a^b f(x)\,dx \ge 0$

If $m \le f(x) \le M$ on $a \le x \le b$ then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

Common Integrals

$\int k\,dx = kx + c$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \ne -1$

$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + c$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$

$\int \ln u\,du = u\ln(u) - u + c$

$\int \mathbf{e}^u\,du = \mathbf{e}^u + c$

$\int \cos u\,du = \sin u + c$

$\int \sin u\,du = -\cos u + c$

$\int \sec^2 u\,du = \tan u + c$

$\int \sec u \tan u\,du = \sec u + c$

$\int \csc u \cot u\,du = -\csc u + c$

$\int \csc^2 u\,du = -\cot u + c$

$\int \tan u\,du = \ln|\sec u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c$

$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + c$

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Standard Integration Techniques

Note that at many schools all but the Substitution Rule tend to be taught in a Calculus II class.

***u* Substitution :** The substitution $u = g(x)$ will convert $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$ using $du = g'(x)\,dx$. For indefinite integrals drop the limits of integration.

Ex. $\int_1^2 5x^2\cos(x^3)\,dx$

$u = x^3 \quad du = 3x^2dx \quad x^2dx = \frac{1}{3}du$

$x = 1 \quad u = 1^3 = 1 \;::\; x = 2 \quad u = 2^3 = 8$

$$\int_1^2 5x^2\cos(x^3)\,dx = \int_1^8 \tfrac{5}{3}\cos(u)\,du = \tfrac{5}{3}\sin(u)\Big|_1^8 = \tfrac{5}{3}(\sin(8) - \sin(1))$$

Integration by Parts : $\int u\,dv = uv - \int v\,du$ and $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$. Choose u and dv from integral and compute du by differentiating u and compute v using $v = \int dv$.

Ex. $\int xe^{-x}\,dx$

$u = x \quad dv = e^{-x} \quad du = dx \quad v = -e^{-x}$

$$\int xe^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + c$$

Ex. $\int_3^5 \ln x\,dx$

$u = \ln x \quad dv = dx \quad du = \frac{1}{x}dx \quad v = x$

$$\int_3^5 \ln x\,dx = x\ln x\Big|_3^5 - \int_3^5 dx = (x\ln(x) - x)\Big|_3^5 = 5\ln(5) - 3\ln(3) - 2$$

Products and (some) Quotients of Trig Functions

For $\int \sin^n x\cos^m x\,dx$ we have the following :

1. ***n* odd.** Strip 1 sine out and convert rest to cosines using $\sin^2 x = 1 - \cos^2 x$, then use the substitution $u = \cos x$.
2. ***m* odd.** Strip 1 cosine out and convert rest to sines using $\cos^2 x = 1 - \sin^2 x$, then use the substitution $u = \sin x$.
3. ***n* and *m* both odd.** Use either 1. or 2.
4. ***n* and *m* both even.** Use double angle and/or half angle formulas to reduce the integral into a form that can be integrated.

For $\int \tan^n x\sec^m x\,dx$ we have the following :

1. ***n* odd.** Strip 1 tangent and 1 secant out and convert the rest to secants using $\tan^2 x = \sec^2 x - 1$, then use the substitution $u = \sec x$.
2. ***m* even.** Strip 2 secants out and convert rest to tangents using $\sec^2 x = 1 + \tan^2 x$, then use the substitution $u = \tan x$.
3. ***n* odd and *m* even.** Use either 1. or 2.
4. ***n* even and *m* odd.** Each integral will be dealt with differently.

Trig Formulas **:** $\sin(2x) = 2\sin(x)\cos(x)$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$, $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

Ex. $\int \tan^3 x\sec^5 x\,dx$

$$\begin{aligned}\int \tan^3 x\sec^5 x\,dx &= \int \tan^2 x\sec^4 x\tan x\sec x\,dx \\ &= \int (\sec^2 x - 1)\sec^4 x\tan x\sec x\,dx \\ &= \int (u^2 - 1)u^4\,du \qquad (u = \sec x) \\ &= \tfrac{1}{7}\sec^7 x - \tfrac{1}{5}\sec^5 x + c\end{aligned}$$

Ex. $\int \frac{\sin^5 x}{\cos^3 x}\,dx$

$$\begin{aligned}\int \frac{\sin^5 x}{\cos^3 x}\,dx &= \int \frac{\sin^4 x\sin x}{\cos^3 x}\,dx = \int \frac{(\sin^2 x)^2\sin x}{\cos^3 x}\,dx \\ &= \int \frac{(1-\cos^2 x)^2\sin x}{\cos^3 x}\,dx \qquad (u = \cos x) \\ &= -\int \frac{(1-u^2)^2}{u^3}\,du = -\int \frac{1-2u^2+u^4}{u^3}\,du \\ &= \tfrac{1}{2}\sec^2 x + 2\ln|\cos x| - \tfrac{1}{2}\cos^2 x + c\end{aligned}$$

Calculus Cheat Sheet

Trig Substitutions : If the integral contains the following root use the given substitution and formula to convert into an integral involving trig functions.

$\sqrt{a^2-b^2x^2} \;\Rightarrow\; x=\frac{a}{b}\sin\theta$	$\sqrt{b^2x^2-a^2} \;\Rightarrow\; x=\frac{a}{b}\sec\theta$	$\sqrt{a^2+b^2x^2} \;\Rightarrow\; x=\frac{a}{b}\tan\theta$
$\cos^2\theta = 1-\sin^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$\sec^2\theta = 1+\tan^2\theta$

Ex. $\int \frac{16}{x^2\sqrt{4-9x^2}}\,dx$

$x=\frac{2}{3}\sin\theta \;\Rightarrow\; dx=\frac{2}{3}\cos\theta\,d\theta$

$\sqrt{4-9x^2}=\sqrt{4-4\sin^2\theta}=\sqrt{4\cos^2\theta}=2\left|\cos\theta\right|$

Recall $\sqrt{x^2}=\left|x\right|$. Because we have an indefinite integral we'll assume positive and drop absolute value bars. If we had a definite integral we'd need to compute θ's and remove absolute value bars based on that and,

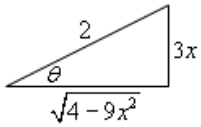
$$\left|x\right| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

In this case we have $\sqrt{4-9x^2}=2\cos\theta$.

$$\int \frac{16}{\frac{4}{9}\sin^2\theta(2\cos\theta)}\left(\tfrac{2}{3}\cos\theta\right)d\theta = \int \frac{12}{\sin^2\theta}\,d\theta$$

$$= \int 12\csc^2\,d\theta = -12\cot\theta + c$$

Use Right Triangle Trig to go back to x's. From substitution we have $\sin\theta=\frac{3x}{2}$ so,



From this we see that $\cot\theta=\frac{\sqrt{4-9x^2}}{3x}$. So,

$$\int \frac{16}{x^2\sqrt{4-9x^2}}\,dx = -\frac{4\sqrt{4-9x^2}}{x}+c$$

Partial Fractions : If integrating $\int \frac{P(x)}{Q(x)}\,dx$ where the degree of $P(x)$ is smaller than the degree of $Q(x)$. Factor denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor in $Q(x)$	Term in P.F.D	Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$	$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Ex. $\int \frac{7x^2+13x}{(x-1)(x^2+4)}\,dx$

$$\int \frac{7x^2+13x}{(x-1)(x^2+4)}\,dx = \int \frac{4}{x-1}+\frac{3x+16}{x^2+4}\,dx$$

$$= \int \frac{4}{x-1}+\frac{3x}{x^2+4}+\frac{16}{x^2+4}\,dx$$

$$= 4\ln\left|x-1\right| + \tfrac{3}{2}\ln\left(x^2+4\right) + 8\tan^{-1}\left(\tfrac{x}{2}\right)$$

Here is partial fraction form and recombined.

$$\frac{7x^2+13x}{(x-1)(x^2+4)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+4} = \frac{A(x^2+4)+(Bx+C)(x-1)}{(x-1)(x^2+4)}$$

Set numerators equal and collect like terms.

$$7x^2+13x = (A+B)x^2+(C-B)x+4A-C$$

Set coefficients equal to get a system and solve to get constants.

$A+B=7$	$C-B=13$	$4A-C=0$
$A=4$	$B=3$	$C=16$

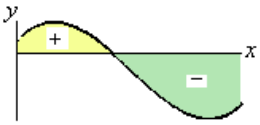
An alternate method that *sometimes* works to find constants. Start with setting numerators equal in previous example : $7x^2+13x = A\left(x^2+4\right)+\left(Bx+C\right)\left(x-1\right)$. Chose *nice* values of x and plug in. For example if $x=1$ we get $20=5A$ which gives $A=4$. This won't always work easily.

Visit http://tutorial.math.lamar.edu for a complete set of Calculus notes.

Calculus Cheat Sheet

Applications of Integrals

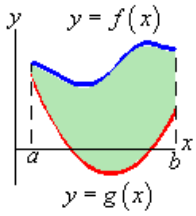
Net Area : $\int_a^b f(x)\,dx$ represents the net area between $f(x)$ and the x-axis with area above x-axis positive and area below x-axis negative.



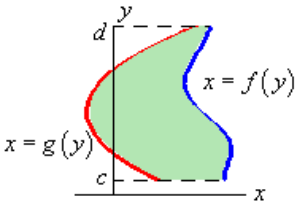
Area Between Curves : The general formulas for the two main cases for each are,

$$y = f(x) \;\Rightarrow\; A = \int_a^b [\text{upper function}] - [\text{lower function}]\,dx \;\;\&\;\; x = f(y) \;\Rightarrow\; A = \int_c^d [\text{right function}] - [\text{left function}]\,dy$$

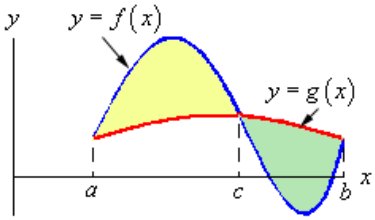
If the curves intersect then the area of each portion must be found individually. Here are some sketches of a couple possible situations and formulas for a couple of possible cases.



$$A = \int_a^b f(x) - g(x)\,dx$$



$$A = \int_c^d f(y) - g(y)\,dy$$

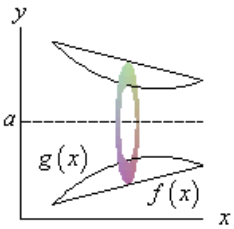


$$A = \int_a^c f(x) - g(x)\,dx + \int_c^b g(x) - f(x)\,dx$$

Volumes of Revolution : The two main formulas are $V = \int A(x)\,dx$ and $V = \int A(y)\,dy$. Here is some general information about each method of computing and some examples.

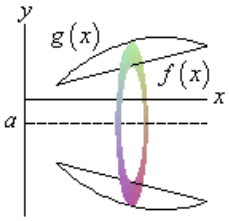
Rings	**Cylinders**
$A = \pi\left((\text{outer radius})^2 - (\text{inner radius})^2\right)$	$A = 2\pi(\text{radius})(\text{width / height})$
Limits: x/y of right/bot ring to x/y of left/top ring	Limits : x/y of inner cyl. to x/y of outer cyl.
Horz. Axis use $f(x)$, $g(x)$, $A(x)$ and dx. Vert. Axis use $f(y)$, $g(y)$, $A(y)$ and dy.	Horz. Axis use $f(y)$, $g(y)$, $A(y)$ and dy. Vert. Axis use $f(x)$, $g(x)$, $A(x)$ and dx.

Ex. Axis : $y = a > 0$



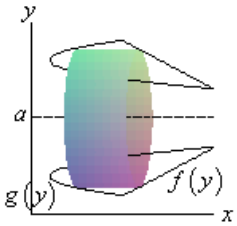
outer radius : $a - f(x)$
inner radius : $a - g(x)$

Ex. Axis : $y = a \le 0$



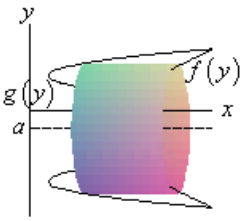
outer radius: $|a| + g(x)$
inner radius: $|a| + f(x)$

Ex. Axis : $y = a > 0$



radius : $a - y$
width : $f(y) - g(y)$

Ex. Axis : $y = a \le 0$



radius : $|a| + y$
width : $f(y) - g(y)$

These are only a few cases for horizontal axis of rotation. If axis of rotation is the x-axis use the $y = a \le 0$ case with $a = 0$. For vertical axis of rotation ($x = a > 0$ and $x = a \le 0$) interchange x and y to get appropriate formulas.

Visit http://tutorial.math.lamar.edu for a complete set of Calculus notes.

Calculus Cheat Sheet

Work : If a force of $F(x)$ moves an object in $a \le x \le b$, the work done is $W = \int_a^b F(x)\,dx$

Average Function Value : The average value of $f(x)$ on $a \le x \le b$ is $f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Arc Length Surface Area : Note that this is often a Calc II topic. The three basic formulas are,

$L = \int_a^b ds$ $\quad SA = \int_a^b 2\pi y\,ds$ (rotate about x-axis) $\quad SA = \int_a^b 2\pi x\,ds$ (rotate about y-axis)

where ds is dependent upon the form of the function being worked with as follows.

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ if $y = f(x),\ a \le x \le b$ $\quad ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ if $x = f(t), y = g(t),\ a \le t \le b$

$ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$ if $x = f(y),\ a \le y \le b$ $\quad ds = \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$ if $r = f(\theta),\ a \le \theta \le b$

With surface area you *may* have to substitute in for the x or y depending on your choice of ds to match the differential in the ds. With parametric and polar you will always need to substitute.

Improper Integral

An improper integral is an integral with one or more infinite limits and/or discontinuous integrands. Integral is called convergent if the limit exists and has a finite value and divergent if the limit doesn't exist or has infinite value. This is typically a Calc II topic.

Infinite Limit

1. $\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$
2. $\int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$
3. $\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$ provided BOTH integrals are convergent.

Discontinuous Integrand

1. Discont. at a: $\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx$
2. Discont. at b : $\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$
3. Discontinuity at $a < c < b$: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ provided both are convergent.

Comparison Test for Improper Integrals : If $f(x) \ge g(x) \ge 0$ on $[a,\infty)$ then,

1. If $\int_a^\infty f(x)\,dx$ conv. then $\int_a^\infty g(x)\,dx$ conv.
2. If $\int_a^\infty g(x)\,dx$ divg. then $\int_a^\infty f(x)\,dx$ divg.

Useful fact : If $a > 0$ then $\int_a^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges for $p \le 1$.

Approximating Definite Integrals

For given integral $\int_a^b f(x)\,dx$ and a n (must be even for Simpson's Rule) define $\Delta x = \frac{b-a}{n}$ and divide $[a,b]$ into n subintervals $[x_0,x_1]$, $[x_1,x_2]$, … , $[x_{n-1},x_n]$ with $x_0 = a$ and $x_n = b$ then,

Midpoint Rule : $\int_a^b f(x)\,dx \approx \Delta x\left[f(x_1^*)+f(x_2^*)+\cdots+f(x_n^*)\right]$, x_i^* is midpoint $[x_{i-1},x_i]$

Trapezoid Rule : $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)++2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right]$

Simpson's Rule : $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$

Visit http://tutorial.math.lamar.edu for a complete set of Calculus notes.

MATHEMATICS DEGREES

Mathematics Degrees

Degree Requirements:

There are two degrees offered:

ASSOCIATE OF SCIENCE DEGREE FOR TRANSFER IN MATH (AS-T DEGREE)

Core Courses (5 courses or 16 units)

Course #	Course Title	Units
MATH 3A	Calculus I	5
MATH 3B	Calculus II	5
MATH 3C	Calculus III	5
	Select one from the following (if you choose both, other courses are optional.)	
MATH 3E	Linear Algebra	3
	or	
MATH 3F	Differential Equations	3
	Select one from the following if necessary to complete 21-23 units for major.	
MATH 11	Discrete Mathematics	4
	or	
MATH 13	Introduction to Statistics	4
	or	
PHYS 4A	General Physics with Calculus	5
	or	
PHYS 4B	General Physics with Calculus	5
	or	
PHYS 4C	General Physics with Calculus	5
	Total Major Units	**21-23**
	IGTEC or CSU GE-Breadth Education Pattern	**37-39**
	CSU Transferrable General Elective Courses to meet 60 units	
	Total Units	**60**

ASSOCIATE OF SCIENCE DEGREE IN MATH (A.S. DEGREE)

Core Courses (5 courses or 16 units)

Course #	Course Title	Units
MATH 3A	Calculus I	5
MATH 3B	Calculus II	5
MATH 3C	Calculus III	5
	Select one from the following (if you choose both, other courses are optional.)	
MATH 3E	Linear Algebra	3
	or	
MATH 3F	Differential Equations	3
	Select one from the following if necessary to complete at least 21 units for major.	
MATH 11	Discrete Mathematics	4
	or	
MATH 13	Introduction to Statistics	4
	Total Major Units	**21-22**
	General Education Requirements	**19**
	Electives	**20**
	Total Units	**60-61**

TUTORING POSITIONS

Working as a tutor in the Laney College Math Lab is great for students who want to master their knowledge of Algebra and Trigonometry. In addition to providing drop-in tutoring to the general Laney student population, the Math Lab also serves those Laney students enrolled in one of four self-paced classes, commonly known as "lab courses": Math 210ABCD (Elementary Algebra), Math 211ABCD (Intermediate Algebra), and Math 220A-G (Technical Math , which includes topics in Algebra, Trig, and Geometry). If you have taken these classes and received good grades (A or B) in them, you may apply to become a tutor in the Lab.

As a tutor, you will

- tutor math
- give out tests and quizzes to students in the Lab
- help people who come into the lab looking for math faculty
- help students with the Math software used in the Algebra courses
- learn how to help students fill out the appropriate paperwork to sign up for classes, and
- do other miscellaneous tasks, such as making copies, running errands, etc.

There is a mandatory training session at the beginning of each semester to get you familiar with the duties expected of you. Also, we expect tutors to come to work **on time** and not to skip work when you have tests or other schoolwork of your own.

The tutors get paid the most that we are allowed to pay students here at Laney and, like any on campus job, the maximum number of hours you can work is 20 hours per week. If you work somewhere else on campus at the same time, you can only have a combined total of 20 hours per week.

To apply, pick up a Tutor Application Form in G201. After you fill out the form, please return it to the Lab Technician (currently Ronald Asseko Messa) in the Math Lab (G201). If the Lab Tech is not there, you can ask someone to put it in the Lab Tech's mailbox. The best times to apply are at the beginning or end of summer. We usually hire people for the Fall Semester who work throughout the academic year. Occasionally we hire tutors mid-semester, but this is rare. However, you can apply at any time. If you apply mid-semester, it is best to check in with the Lab Tech at the end of the semester to make sure that we still have your paperwork

CAMPUS RESOURCES

The Laney College Student Services mission is to provide quality support services with an appreciation for diversity by empowering students to achieve their academic, career, and personal goals.

Below is information on the different student services resources available to students at Laney. (Retrieved from https://laney.edu/student-services/)

Adult Transitions Program
Tower Building, Room T-214
(510) 464-3398
laney.edu/adulttransitions/

Laney Adult Transitions Program serves adults (18+), who are interested in enrolling at Laney campus to further their education and increase their employability. Laney is a safe space that welcomes diversity of all kinds and fosters respect and indiscriminate engagement amongst staff and students.

Assessment Center
Building A, Room A-101
(510) 464-3515
laney.edu/assessment_center
laneyassessment@peralta.edu

The Assessment Center is designed to provide a number of services to students and members of the community:

1. Orientation to the college.
2. Basic skills assessment in language usage, reading, and mathematics to assist students in selecting courses appropriate to their current skill levels and/or for use in consultation with counselors.
3. ESL (English as a Second Language) assessment tests.

Athletics
Laney Field House
(510) 464-3478
laney.edu/athletics

The Laney College intercollegiate athletic program provides students the opportunity to participate in men's football, baseball, and women's basketball, swimming, track and field, volleyball and water polo. Laney College is a member of the Bay Valley Conference (BVC) and the California Community Colleges Athletic Association (CCCAA).

Bookstore
Student Center, Lower Level
(510) 464-3514

laney@bkstr.com
laney.edu/bookstore/

The Laney College Bookstore carries all course materials required for courses at the College, as well as school supplies, imprinted clothing and gift items, course-related reference materials, beverages, and snacks.

Cooperative Agencies Resources for Education (CARE)
Building A, Room A-106
(510) 464-3423
laney.edu/eops
laneycollegeeops@peralta.edu

Cooperative Agencies Resources for Education (CARE) is an educational program which represents a cooperative effort between the community college, Department of Social Services, and the Employment Development Department to help single parents achieve educational goals. CARE offers the following services: counseling, orientation, personal development classes, support groups, peer advising, workshops, financial, and childcare assistance.

Child Care
East 10th St. & 2nd Ave.
(510) 464-3574
laney.edu/child_care/

Laney College provides free, limited child-care services for pre-school children of qualified full-time students. The Children's Center is administered by the Peralta Community College District. Students who wish to enroll their children in the campus Children's Center should apply directly at the Center.

Counseling
Tower Building, Room T-301
(510) 464-3152 or 3154
laney.edu/counseling
laneycounseling@peralta.edu

Laney College offers a wide range of professional counseling services for enrolled students and prospective students. These services include:

1. Educational planning for degrees, occupational certificates, and transfer to four-year institutions
2. Evaluation of transcripts for graduation and transfer
3. Career counseling
4. Help in developing good study skills
5. Advising on academic problems
6. Personal counseling and referral to off-campus services
7. Classes and special workshops; e.g., career and life planning

The College requires all new students to participate in the Student Success Matriculation Program. Students must develop their first year "Educational Plan" with a counselor.

Disabled Students Programs And Services (DSPS)
Building E, Room E-251
(510) 464-3428
laney.edu/dsps
cwchan@peralta.edu

The Disabled Students Programs and Services (DSPS) provide assistance designed to facilitate equal educational opportunities for students with disabilities. In order to obtain support from this department, students must be enrolled at Laney College and provide documentation of their disability. The programs to assist students include:

1. The Disability Resource Center: offers services according to individual needs and availability of resources. Services include, advocacy, disability related counseling, vocational and educational planning, and individually prescribed support services, such as test accommodations, mobility orientation, readers and referrals for special instruction.
2. Hearing Impaired Services provides interpreting, registration, and other support services for hearing impaired students.
3. Alternative Media Center provides books and other materials in other formats such as Braille, e-text, and large print.
4. Workability III, a joint program with California Department of Rehabilitation, helps students find work in their chosen field.
5. DSPS sponsors specialized educational programs as follows:

    a. The Learning Skills Program – for students with learning disabilities, offering specialized tutoring and test accommodations for Laney classes.
    b. The High Tech Center – Adapted computer technology training and computer aided instruction for students with disabilities as referred by a DSPS Counselor, using state-of-the art hardware and software. Laboratory time is available for trained students to use the computers for class assignments.

Courses offered through the Disabled Students Programs and Services may be repeated for an unlimited number of times based on:

1. Need for adequate preparation for other courses
2. Enhanced learning and continuing success in academic areas
3. Measurable progress

For these programs or services, appointments must be made with a DSPS Counselor in the Disability Resource Center.

Emergency Resources for Students
This document (https://laney.edu/tcamp/wp-content/uploads/sites/445/2016/03/Emergency-Resources-Spring-2016.pdf) lists information on housing for the homeless, food banks, domestic

violence, and other information. For Fall 2017 Laney Food Bank information, Please visit https://laney.edu/wp-content/uploads/2017/09/Laney-Food-Bank.jpg for more information.

Employment Services Center
Tower Building, 2nd Floor, Room T-201
(510) 464-3352
laney.edu/employment/

The Employment Services Center assists students with a number of services, including resume building, interview workshops, and employment information. The center also holds numerous job fairs each semester to connect students with potential employers.

Extended Opportunity Programs & Services (EOPS)
Building A, Room A-106
(510) 464-3423
laney.edu/eops
laneycollegeeops@peralta.edu

The Extended Opportunity Programs and Services (EOPS) provide educational opportunity for non-traditional students who are educationally and economically disadvantaged. Support services include, registration assistance, orientation, counseling, peer advising, tutoring, transfer assistance, tuition fee waiver for CSU and UC transfers, book vouchers, and financial assistance for eligible full-time students.

Financial Aid
Building A, Room 101
(510) 464-3414
laney.edu/financial_aid

International Affairs & Distance Education
333 E.8th St., Oakland, CA
(510) 587-7834
http://international.eperalta.org/

The Office of International Affairs and Distance Education provides quality support services to enhance student learning for international students in the following areas: admissions, immigration issues, academic/personal counseling and advising, orientation for new students, tuitions issues, housing, activities, trips, health, medical, and safety issues, tax workshops, assistances with Social Security and Department of Motor Vehicle matters, as well as online support services.

All international students must first apply through the Office of International Affairs and Distance Education by completing and submitting the International Student Application Form, along with the $50 application fee, before enrolling at the colleges. The application may be downloaded at http://international.eperalta.org/.

Additionally, all new international students are required to attend a mandatory orientation held at the start of each semester. Students will receive information regarding academic matters, immigration issues, heath/safety issues, and much more.

Learning Communities
For detailed information on learning communities, please visit https://laney.edu/learning_communities/

Library/Learning Resources Center
L Building
(510) 464-3497
laney.edu/wp/library

The Library and Learning Resources Center (Building L) houses a wide variety of services and resources for students, faculty and staff.

The Library, on the main floor, contains more than 30,000 volumes, 215 periodicals, pamphlets and microform materials. Also available are photocopiers and computers for library research, study rooms and a leisure reading area.

Newspaper: Laney Tower
Tower Building, 7th floor
(510) 464-3459
laneytower.com/

The Laney Tower is produced by students in Laney's Journalism Department. The Tower, first published in 1952, has won many awards over the years and has trained many professional journalists. To join the newspaper staff, students should contact the Journalism Department.

Phi Theta Kappa
Brandi Howard, Advisor
Tower Building, T-804
(510)-464-3219
laney.edu/phi_theta_kappa/

Phi Theta Kappa, an international community college honor society, promotes scholarship, service, and community leadership. Laney's chapter, Alpha Chi Theta was founded in May 1992. Chapter members participate in numerous campus and community projects.

Membership is open to all students who have accumulated 12 semester units with a GPA of 3.50 or higher.

Safety Aide Program
(510) 464-3126
laney.edu/safetyaides
laneysafetyaides@peralta.edu

The Laney Safety Aides are members of a team of diverse student leaders in good academic standing who have demonstrated excellent leadership abilities. Safety Aides reinforce safety policies here on campus.

While working closely with staff, faculty, and law enforcement, the Safety Aides help the Laney campus continue to maintain a safe learning environment. Safety Aides assist with patrolling the campus, providing escorts to BART and the parking lots surrounding our campus. Through their hard work, safety aides develop important skills including, leadership, communication skills, safety policies and procedures, and they are CPR compliant by completing a 40-hour training session prior to being hired.

Scholarships and Awards
Tower Building, T-813
Office of the Vice President of Student Services
(510) 464-3244
laney.edu/studentservices/scholarships/

The College maintains a limited listing of scholarships and awards sponsored by various alumni, professional groups, and other friends of the College. Most scholarships are special merit awards used to give recognition to individuals who have distinguished themselves in areas of academic performance and co-curricular activities and who demonstrate financial need.

Scholarship announcements are posted around the campus and on the Laney Scholarship webpage. Information is available at the Office of the Vice President of Student Services.

Student Activities
Student Center, Fourth Floor, Rm. 412
(510) 464-3536
laney.edu/student_activities/

Student activities are recognized as an integral part of the college curriculum. These activities provide students with opportunities to apply concepts learned in class while continuing to build skills for careers, transfer, and community involvement. Students interested in forming new clubs, running for student body office, or other student activities should contact the Student Activities Advisor.

Student Ambassadors
Welcome Center, Building A, Room A-109
(510) 464-3122
laney.edu/outreach/ambassadors/

Student Ambassadors are members of a team of diverse student leaders in good academic standing who have demonstrated leadership abilities, communication skills and event coordination. Working closely with staff and faculty, student ambassadors represent the college both on and off campus, attending educational events, conducting tours and assisting new and returning students with the enrollment and orientation process.

Student Government: Associated Students Of Laney College (ASLC)
Student Center, Fourth Floor, Rm. 412
(510) 464-3536
laney.edu/studentgovernment

Officers elected by the student body comprise the Student Council and Senators who serve as the governing body for student affairs. Council meetings are open to all interested students. The Council and its committees provide an opportunity for students to assume leadership roles and actively participate in student activities.

The Associated Students of Laney College charters all student organizations. These organizations must submit a constitution approved by the ASLC. The clubs are governed by the Interclub Council. Any group of students having a common interest may petition the Student Council for recognition as a chartered club. The club must have a faculty sponsor and adhere to the general rules and regulations established by the Student Council and the College administration. All activities and events sponsored by student groups must be supervised by members of the faculty or staff.

A club or organization chartered by the Student Council must abide by policies set forth by the College. These policies require open membership and prohibit hazing or secret initiations. For more information, visit http://www.laney.edu/student_activities/

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Transfer Center
Tower Building, Room T-201
(510) 464-3135
laney.edu/transfer/

The Laney College Transfer Center offers a variety of services to help students transfer to the California State University and University of California system, and independent colleges and universities including Historically Black Colleges. The Transfer Center offers:

• Supportive Resources & Services: Library of college handbooks and reference handbooks, four-year college representative visits, transfer workshops and information sessions, computers for research and applications, field trips to four-year colleges, online calendar of transfer activities, and an annual Transfer Day fair.

• Transfer Admission Guarantee (TAG) is a program that offers students guaranteed admissions to participating colleges and universities. Several University of California campuses offer guaranteed admissions to students who meet specific requirements.

Tutoring Centers

Tutoring at Laney College is offered in three main strategically-located centers on campus:

James Oliver Community Writing Center
Tutoring in English and English for Speakers of Other
Languages (ESOL)
Building B, Room B-260
(510) 464-3426
Contact: Pam Crumpton, pcrumpton@peralta.edu
laney.edu/wp/writingcenter

Math Lab
(510) 464-3448
Building G, G-201
Contact: Ronald Asseko Messa
rmessa@peralta.edu or laney-mathlab@peralta.edu

Tutoring Resource Center
Subjects include: Chemistry, Physics, Chinese, French, and Spanish
Biology tutoring is available in the Bio Lab (B-202)
Eagle Village (EV1)
Contact: Kevin Wade
kwade@peralta.edu

The Laney College Tutor Program provides instructional support across the college curriculum. Current operations include the Math Lab, James Oliver Community Writing Center, and the Tutoring Resource Center.

The Math Lab offers drop-in tutoring in mathematical concepts, a productive study environment, access to instructors, and paid work experience for student tutors.

The Writing Center provides drop-in tutoring in reading and writing across the curriculum, Writing Workshop courses in ESL and English, access to computers for use in completion of writing assignments, and paid work experience for student tutors.

The Tutoring Resource Center provides assistance in Biology, Chemistry, Chinese, French, Physics, and Spanish.

Students interested in working as tutors in the Tutor Program should enroll in LRNRE 30 Introduction to Tutoring and contact the tutoring coordinator. Tutoring in other subjects is offered through specific departments in designated areas. Check with your instructors for current availability.

Veterans Affairs
Student Center, 3rd Floor, Room 300
(510) 986-6994
laney.edu/veteran_affairs
jcarey@peralta.edu

Laney College is approved by the Council for Private Postsecondary and Vocational Education, and Veterans Administration as a degree granting institution for veterans and eligible dependents seeking educational or vocational training under Title 38, United States Code.

Procedures for applying and certifying veterans' benefits are provided by the Veterans Affairs Office in the Student Center.

To receive benefits all veterans are required to consult with a counselor for development of an Educational Plan as mandated by the Veterans Administration. Non-degree college credit is given for completed courses numbered 250 and higher.

Veterans requesting credit for military experience or courses taken during military service may receive six elective units toward their associate degree. All veterans not enrolled in the veterans program and who have completed 12 semester units may obtain military credit by providing a copy of their discharge papers (DD-214) to the Admissions and Records Office on campus.

Failure to take the proper classes can result in an overpayment and the reduction or termination of benefits.

Standards of Progress for Veterans Receiving Educational Benefits
A veteran student who is on Academic Probation for two (2) consecutive semesters shall be subject to discontinuance of benefits if the student earned a grade-point average of less than 2.0 ("C" grade). This directive is separate and apart from Laney College's Standards for Academic Dismissal.

Welcome Center
Building A, Room A-109
(510) 464-3540
laney.edu/welcome_center/
cbradford@peralta.edu

Open for students to receive assistance with online admissions, registration, financial aid application (FAFSA) processes, photo ID services, and class schedule print outs are also available.

Wellness Center I
Tower Building, T-250
(510) 464-3384
laney.edu/health_center

Wellness Center II
Student Center, 4th Floor, Room SC-410
(510)464-3134
laney.edu/health_center

Laney provides on-campus services of a nurse and mental health counselor to support student health and wellness. Consultation and health services are free of charge to all Laney College students.

Services Available:

• Behavorial and Mental Health Counseling
• Birth control
• Family planning
• Health education and referral services
• HIV testing
• On-sight enrollment into public health insurance programs
• Physical exams
• Pregnancy testing
• Pap smears
• Sexually transmitted infection testing and treatment
• TB testing (Mondays only)
• Vaccinations, flu shots (seasonal), whooping cough (pertussis) immunizations
• Urgent care clinic referrals

(Medi-Cal, Family PACT)
Please note: Labwork, pharmacy, and x-ray services are NOT provided at the Peralta Wellness Center. If you have no insurance coverage, you will be charged for these services.

Workforce Development/ CalWORKs Program
Building A, Room A-106
(510) 986-6946
laney.edu/calworks/

CalWORKs funds are for the purpose of assisting welfare recipient students and those in transition off of welfare to achieve long-term self-sufficiency through coordinated student services offered at community colleges including: work study, job placement, child care, coordination, curriculum development and redesign, and under certain conditions post-employment skills training, and instructional services.

DIRECTORY OF CAMPUS ADMINISTRATORS

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Dean of Mathematics and Sciences

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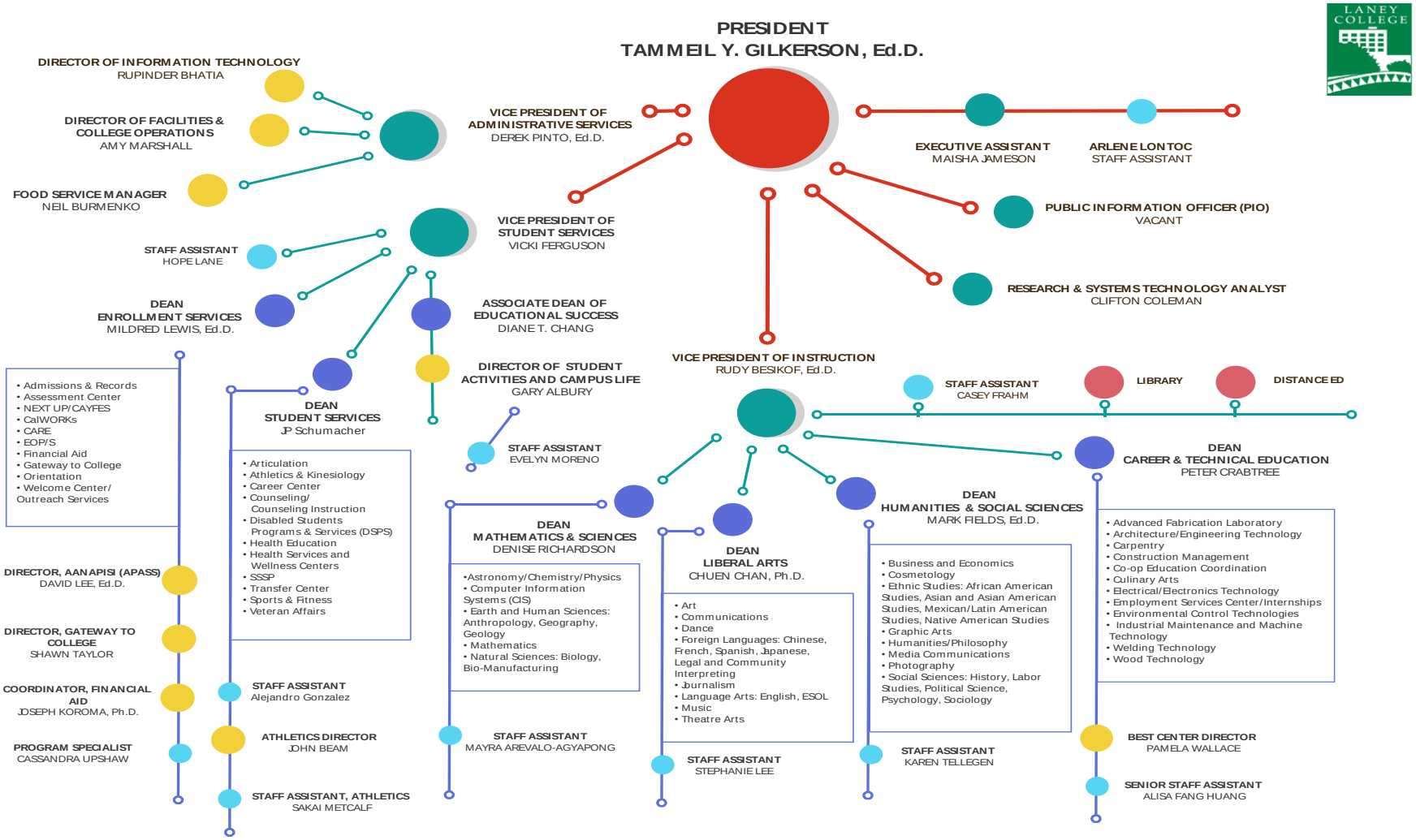
Director of AANAPISI (APASS)

Mildred Lewis, Ed.D.
mildredlewis@peralta.edu
Dean of Enrollment Services

Laney College Organizational Chart

2019- 2020

Dream. Flourish. Succeed.



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SPECIAL THANKS:

- **DENISE RICHARDSON**
  DEAN OF MATHEMATICS AND SCIENCES
- **MAYRA AREVALO**
  STAFF ASSISTANT MATHEMATICS AND SCIENCES
- **DAVID ROSS**
  DEPARTMENT CHAIR MATHEMATICS
- **RONALD ASSEKO MESSA**
  INSTRUCTIONAL ASSISTANT MATHEMATICS
- **MATHEMATICS TUTORS IN MATH LAB**
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  CALVIN HSIEH
  MARK DITTMER
  MATTHEW ZUBERBUHLER
  VERONICA FARIAS
  YANGLIN LIU
  YUYANG XING

SOURCES:

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