Making Measurements and Reading Graphs - Interpolation

Anytime we measure things in the laboratory or read values from a graph, we will be following these guidelines:

- Estimate between the lines.
- Include all certain digits and the first uncertain digit. (This gives us more information that just rounding off the number.)
- Estimate to the nearest one-tenth of the smallest division.
- Sometimes the divisions are so close together that it makes it difficult to read between the lines, but do your best.

Here are the steps:

- First, examine the measuring device or scale or numbers along the graph. Figure out the value of the smallest division.
- Then, divide that value by 10. You will read between the lines to the nearest tenth of the smallest division.
- Read and record the value of the measurement, including units, while estimating between the lines.
- Make sure to double-check your measurement by looking again at the scale of the device or graph - does your result make sense?

Determining the Uncertainty

If the smallest division is 1 mL, you will estimate between the lines to the nearest 0.1 mL (because 1 mL ÷ 10 = 0.1 mL). Your measurement's uncertainty will be ± 0.1 mL, and you will write the number with one decimal place (one digit after the decimal). [For example, the measurement might be 15.7 mL or 2.8 mL or 24.0 mL. All of these have one decimal place, which means they have been measured to the nearest ± 0.1 mL.]

If the smallest division is 0.1 g, you will estimate between the lines to the nearest 0.01 g (because 0.1 g ÷ 10 = 0.01 g). Your measurement's uncertainty will be ± 0.01 g, and you will write the number with two decimal places (two digits after the decimal). [For example, the measurement might end up being 245.15 g, or 5.93 g, or 27.30 g, or anything with two digits after the decimal.]

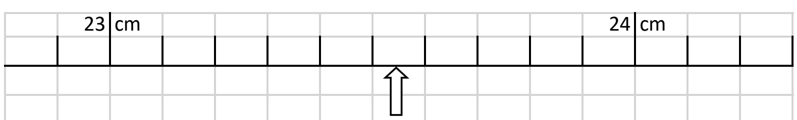
Notice that both the measurement and the uncertainty have the same units. These units should be included for both.

If the smallest division is 0.2 cm, then you would estimate to the nearest ± 0.02 cm. In addition, you would need to make sure that your measurement has two decimal places, and that the final digit is an even number. (If it doesn't end in an even number, then that means that you didn't estimate to the nearest 0.02 cm, but probably to the nearest 0.01 cm... and it's probably not possible to be that precise with the equipment or graph you were using.) [For example, the measurement might be 46.94 cm, or 6.38 cm, or 35.70 cm... or anything that has two decimal places in which the last digit is even.]

If the smallest division is 0.05 g, you would estimate to the nearest ± 0.005 g. Your result should have 3 digits after the decimal, and it should end in either 0 or 5. Otherwise, your measurement will not match the uncertainty of your equipment, so it is less trustworthy. [Examples that would be possible in this case: 4.835 g, 12.930 g, 0.075 g, 0.860 g, etc. All of these have three decimal places and their last digit is either 0 or 5.]

Examples of Interpolation

Example 1



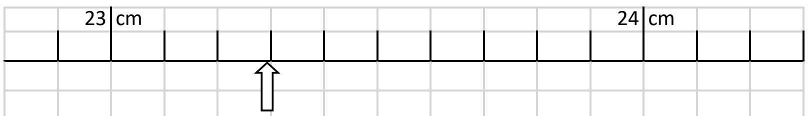
First, look at the numbers on the scale and the number of divisions.
In this case, there are 10 divisions between 23 cm and 24 cm. This means that each division is 0.1 cm (because 24-23 = 1 cm, and 1 cm ÷ 10 divisions = 0.1 cm per division).

We therefore need to estimate between the lines to the nearest 0.01 cm. This is because we can estimate to the nearest one-tenth of the smallest division, and 0.1 cm ÷ 10 = 0.01 cm. Anything we estimate from this scale should have 2 decimal places.

Now, let's look at the arrow. The arrow is between the 23.5 and the 23.6 cm marks.
To determine the estimated digit, imagine (but don't try to write down) this smallest division being divided into 10 smaller divisions.
This arrow is not quite halfway between 23.5 and 23.6. If it was halfway between them, the measurement would be 23.55 cm. Because this is close to halfway but not quite halfway, I would estimate the last digit as a 4. The measurement would be 23.54 cm.
Note that 23.53 cm might also be an acceptable value - sometimes measurements vary by one in the last digit.

Note that 23.52 would not be an acceptable estimation of this measurement, since the arrow is definitely beyond 1/5 of the way between 23.5 and 23.6 cm.
23.55 would also not be acceptable, because it is definitely less than halfway between 23.5 and 23.6 cm.
Answer: 23.54 cm.

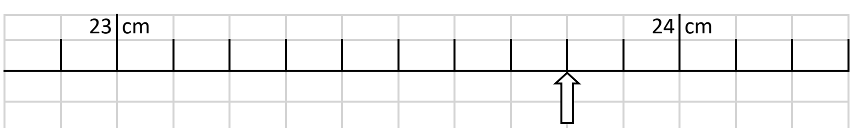
Example 2



This example uses the same scale as the previous one. We already know that the divisions are 0.1 cm and we will interpolate to the nearest 0.01 cm.

For this measurement, the arrow is between the 23.2 and the 23.3 marks, and it is very close to the 23.3 mark. I would estimate this value as 23.29 cm. It is definitely not on the 23.3 line, so I would not round it to 23.3 or 23.30 cm. Also, it is too close to the 23.3 line to be read as 23.28 cm. (In order to measure as 23.28, the arrow would have to be four-fifths of the way between the 23.2 and the 23.3 marks.)
Answer: 23.29 cm.

Example 3

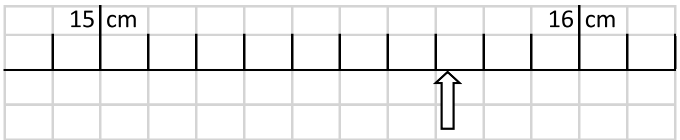


Again, we are using the same scale as before. This time, the arrow appears to be pointing directly at the 23.8 cm line. Because it is right on the line, and because we are measuring this to the nearest 0.01 cm, our measurement would have to be 23.80 cm.

Note that expressing the measurement as 23.8 cm would be incorrect. Writing 23.8 cm as the result would imply that you only measured the value to the nearest 0.1 cm, and that the true value could be between 23.7 and 23.9 cm. It would imply fewer significant figures and a larger uncertainty than there is in reality.

Answer: 23.80 cm

Example 4



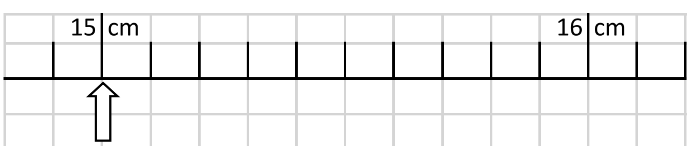
First, look at the scale and the number of divisions. There are 10 divisions between 15 cm and 16 cm, so each division is 0.1 cm, as before. We need to estimate to the nearest 0.01 cm.

Then, look at what the arrow is between. This one is between 15.7 cm and 15.8 cm. It is less than halfway between these markings. It is not super close to 15.7 cm. It appears that it is approximately 1/5-of the way between the markings. I would estimate it as 15.72 cm.

It would not be 15.71 cm, because it isn't close enough to the 15.7 mark. It would not be 15.75 cm, because it is not halfway between the marks. It is not 15.74 cm, because it would have to be closer to halfway but slightly less. It might be all right to record it as 15.73 cm, since it's a little hard to tell whether it is 2/10 of the way between the marks or 3/10 of the way between them.

Answer: 15.72 cm (or 15.73 cm).

Example 5



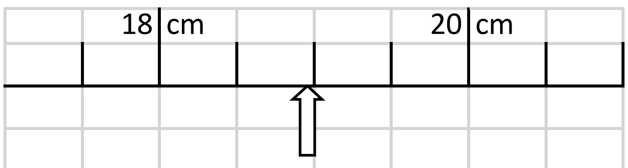
Here, the divisions are every 0.1 cm, and we can read to the nearest 0.01 cm.
This time, the arrow points directly at the 15 cm mark. This means our last digit will end in zero. Because we are measuring to the nearest 0.01 cm, our measurement would have to be stated as 15.00 cm. Notice that in this case we are adding two zeros to the end of the measurement.

It would not be correct to report 15 cm, because this would mean that it was measured to the nearest 1 cm, and the true value could be between 14 and 16 cm.
It would also not be correct to report it as 15.0 cm, because this means that it was measured to the nearest 0.1 cm and the true value could be between 14.9 cm and 15.1 cm.

This is clearly not what we did - you can tell when you look at the measuring scale we used. Our measurement is 15.00 cm, and the true value could be between 14.99 cm and 15.01 cm.

Answer: 15.00 cm.

Example 6



This one is less straightforward.

First, look at the numbers and the number of divisions. Between 18 and 20 cm, there are four divisions. (20 cm - 18 cm = 2 cm, and 2 cm ÷ 4 divisions = 0.5 cm per division.) This means each division is 0.5 cm. We can read this scale to the nearest 0.05 cm (this is one tenth of the smallest division).

Again, when we read between the lines, we imagine 10 smaller divisions between the smallest division shown. But here, each of the imaginary smaller divisions will be 0.05 cm. This will require some extra thinking.

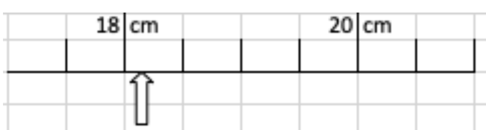
For this measurement, the arrow is almost to the 19 mark (which is the one halfway in between the 18 cm and the 20 cm marks). It is between the 18.5 and the 19.0 marks, but very close to the 19.0 mark. It looks like it is either 9/10 or 8/10 of the way between these markings. If we read it as 9/10 of the way between 18.5 and 19.0, then the measurement would be 18.95 cm. (Because 9/10 × 0.5 cm = 0.45, and 18.5 + 0.45 = 18.95 cm.) If we read it as 8/10 of the way between these markings, it would be read as 18.90 cm.
Note that our answer must end in either 0 or 5.
Answer: either 18.95 cm or 18.90 cm.

Example 7

Again, between 18 cm and 20 cm there are four divisions, so each division is 0.5 cm, and we need to estimate between the lines to the nearest 0.05 cm.

Here the arrow is between the 19.0 cm and the 19.5 cm mark.
The arrow is definitely more than halfway between these markings.
It looks to me that it is about 7/10 of the way between the markings. It doesn't look quite close enough to halfway to be considered 6/10 of the way (but maybe). It doesn't look close enough to the 19.5 marking to be considered 8/10 of the way.
So if it is 7/10 of the way between the 19.0 and the 19.5 marking, then that would be 7 × 0.05 = 0.35, so this added to 19.0 would be 19.35 cm. (Another way of thinking about it: 7/10 of 0.5 (the size of the division) would be 0.35, so 19.0+0.35 = 19.35 cm.)
Note that this measurement ends in a 5 or a 0.
Answer: 19.35 cm.

Example 8

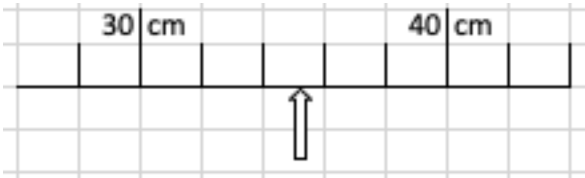


Each division is 0.5 cm. Measure to nearest 0.05 cm. Last digit ends in 0 or 5.

Here, the arrow appears to be 2/10 or 3/10 of the way between the 18.0 and the 18.5 cm marks. It looks to me that it's closer to 3/10 of the way.
(3/10) of 0.5 cm = 0.15 cm. 18 cm + 0.15 cm = 18.15 cm.
If it's 2/10 of the way, then 2/10 x 0.5 cm = 0.10 cm, and 18 + .10 cm = 18.10 cm.

Answer: 18.15 cm or 18.10 cm.

Example 9

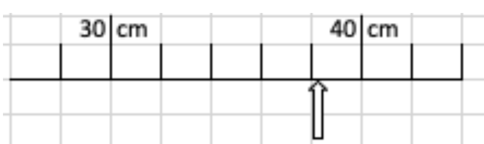


Examine this scale. there are 5 divisions between 30 and 40 cm. 40 - 30 = 10 cm. 10 cm ÷ 5 divisions = 2 cm per division.
We will estimate between the lines to the nearest ± 0.2 cm. (This is 1/10 of the smallest division.)
The measurement should end in an even digit (0, 2, 4, 6, or 8).

The arrow is between the 34 and the 36 cm marks. The arrow is more than halfway between these marks. It appears to be 6/10 of the way between the marks.
6/10 of 2 cm is 1.2 cm. 34 + 1.2 cm = 35.2 cm.

Answer: 35.2 cm.

Example 10



This one has the same scale as the previous one.
Each division is 2 cm, and we read between the lines to the nearest 0.2 cm.

Here, the arrow is between the 38 and the 40 cm marks. It is very close to the 38 mark.
It appears to be either 2/10 or 1/10 of the way between the marks.
If it is 2/10, then 2/10 × 2 cm = 0.4 cm.   38 + 0.4 cm = 38.4 cm.
If it is 1/10 of the way, then 1/10 × 2 cm = 0.2 cm. 38 + 0.2 cm = 38.2 cm.

Answer: 38.4 cm or 38.2 cm.