## Logarithm Math and $\mathrm{p} K_{\mathrm{a}}$

Definitions:
log means $\log _{10}$, or "base 10 log"
$\ln$ (or natural log) means $\log _{\mathrm{e}}$, or "base $e$ log" where $e \sim 2.718 \ldots$
For base 10 log,

$$
\log \left(10^{x}\right)=x \text { and } 10^{\log x}=x \quad\left(\text { e.g., } \log (2365)=3.374 \text {, so } 10^{3.374}=2365\right)
$$

Similarly, for base $e$ log,

$$
\ln \left(e^{x}\right)=x \text { and } e^{\ln x}=x \quad\left(\text { e.g., } e^{2.453}=11.623, \text { so } \ln (11.623)=2.453\right)
$$

In Chemistry,
$\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{H}^{+}\right]=$hydronium ion (hydrogen ion) concentration in terms of molarity ( M , or mol/L)
$\mathrm{pH}=-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log \left[\mathrm{H}^{+}\right]=$negative log base 10 of $\left[\mathrm{H}^{+}\right]$
If $\left[\mathrm{H}^{+}\right]=1.0 \times 10^{-3} \mathrm{M}$, then $\mathrm{pH}=3.0$ (and is a "unitless" number)
If $\mathrm{pH}=6.4$, then $\left[\mathrm{H}^{+}\right]=1.0 \times 10^{-6.4} \mathrm{M}$ (a very small concentration)
$K_{\mathrm{a}}$, the acidity constant, is defined as:
$K_{\mathrm{a}}=\frac{\left[\mathrm{H}^{+}\right]\left[\mathrm{A}^{-}\right]}{[\mathrm{HA}]}$ and describes the dissociation for an acid HA shown below:

$\mathrm{p} K_{\mathrm{a}}=-\log \left(K_{\mathrm{a}}\right)$ and is a useful number to describe the acidity of any acid, especially weak ones.
Acetic acid $\left(\mathrm{CH}_{3} \mathrm{CO}_{2} \mathrm{H}\right)$ has a $\mathrm{p} K_{\mathrm{a}}$ of 4.74 so the value of $K_{\mathrm{a}}$ is $10^{-4.74}=1.82 \times 10^{-5}$ and is a weak acid.
Negative $\mathrm{p} K_{\mathrm{a}}$ means a strong acid (e.g., $\mathrm{H}_{2} \mathrm{SO}_{4}$ has $\mathrm{p} K_{\mathrm{a}}=-3$ )

Question 1: How much stronger an acid is trifluoroacetic acid (TFA), $\mathrm{p} K_{\mathrm{a}} 0.23$, compared to acetic acid $\left(\mathrm{CH}_{3} \mathrm{CO}_{2} \mathrm{H}\right), \mathrm{p} K_{\mathrm{a}} 4.74$ ?

## Answer:

The difference in the $\mathrm{p} K_{\mathrm{a}}$ 's shows the magnitude.
$4.74-0.23=4.51$ "orders of magnitude," so TFA is $10^{4.51}$ (or 32,400 ) times more acidic than HOAc
Question 2: Derive the Henderson-Hasselbach equation $\mathrm{pH}=\mathrm{p} K_{\mathrm{a}}+\log \left[\mathrm{A}^{-}\right] /[\mathrm{HA}]$ from the definition of $K_{\mathrm{a}}$.

