Numbers (Part I)

1. The equation $a^3+b^3+c^3=2008$ has a solution in which a, b, c are distinct even positive integers. Find $a+b+c$. [2008S, 22]
2. Each bag to be loaded onto a plane weighs either 12, 18, or 22 lb. If the plane is carrying exactly 1000 lb of luggage, what is the largest number of bags it could be carrying? [2008S, 82]
3. Call a positive integer *biprime* if it is the product of exactly two distinct primes (thus 6 and 15 are biprime, but 9 and 12 are not). If N is the smallest number such that N, $N+1$, $N+2$ are all biprime, find the largest prime factor of $N(N+1)(N+2)$. [2008S, 17]
4. Let r, s, t be nonnegative integers. For how many such triples (r,s,t) satisfying the system $\begin{cases} rs+t &= 24 \\ r+st &= 24 \end{cases}$ is it true that $r+s+t=25$? [2008S, 26]
5. The digits 1 to 9 can be separated into 3 disjoint sets of 3 digits each so that the digits in each set can be arranged to form a 3-digit perfect square. Find the last two digits of the sum of these three perfect squares.
   A. 26 B. 29 C. 34 D. 46 E. 74
   [2008S, E]
6. The sequence $\{a_n\}$ is defined by $a_0=a_1=a_2=1$, and $a_{n-3}a_n-a_{n-2}a_{n-1}=(n-3)!$ for $n\geq 3$. If 5^k is the largest power of 5 that is a factor of $a_{100}a_{101}$, find k. [2008S, 24]
7. Trina has two dozen coins, all dimes and nickels, worth between $1.72 and $2.11. What is the least number of dimes she could have? [2007F, 11]
8. Replace each letter of AMATYC with a digit 0 through 9 to form a six-digit number (identical letters are replaced by identical digits, different letters are replaced by different digits). If the resulting number is the largest such number which is a perfect square, find the sum of its digits (that is, $A+M+A+T+Y+C$) [2007F, 36]
9. Add any integer N to the square of $2N$ to produce an integer M. For how many values of N is M prime? [2007F, 2]
10. When certain proper fractions in simplest terms are added, the result is in simplest terms: $\frac{2}{15}+\frac{1}{21}=\frac{19}{105}$; in other cases, the result is not in simplest terms: $\frac{2}{15}+\frac{5}{21}=\frac{39}{105}=\frac{13}{35}$. Assume that $\frac{m}{15}$ and $\frac{n}{21}$ are positive proper fractions in simplest terms. For how many such fractions is $\frac{m}{15}+\frac{n}{21}$ not in simplest terms? [2007F, 48]
11. Let r, s, and t be nonnegative integers. How many such triples (r,s,t) satisfy the system $\begin{cases} rs+t=14 \\ r+st=13 \end{cases}$? [2007F, 2]
12. If AM/AT = .YC, where each letter represents a different digit, AM/AT is in simplest terms, and A $\neq$ 0, then AT =? [2007S, 25]
13. Two adjacent faces of a rectangular box have areas 36 and 63. If all three dimensions are positive integers, find the ratio of the largest possible volume of the box to the smallest possible volume. [2006F, 9]

14. In the expression (AM)(AT)(YC), each different letter is replaced by a different digit 0 to 9 to form three two-digit numbers. If the product is to be as large as possible, what are the last two digits of the product?
A. 20 B. 40 C. 50 D. 60 E. 90 [2006F, B]
15. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 5 & -10 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the smallest possible value of $a+b+c+d$, if a, b, c, and d are all positive integers. [2006F, 16]
16. The year 2006 is the product of exactly three distinct primes p, q, and r. How many other years are also the product of three distinct primes with sum equal to $p+q+r$? [2006F, 4]
17. How many positive integers less than 1000 are relatively prime to 105? Two integers are relatively prime if their greatest common divisor is 1. [2006F, 457]
18. How many 4-digit numbers whose digits are all odd are multiples of 11? [2006F, 85]
19. Find the tens digit of 3^{2007}. [2006F, 8]
20. In the sequence a_1, a_2, a_3, …, $a_1 = 1$, $a_2 = 2$, $a_3 = 5$, and for all $n \geq 3$, $a_{n-1}a_{n-2} = 2a_n a_{n-2} - 2a_{n-1}a_{n-1}$. Find a_{2006}/a_{2005}. [2006F, 1004]