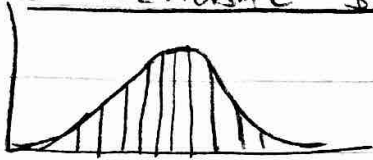


MODULE 17: THE NORMAL DISTRIBUTION 3-11-19
 (UNIMODAL & SYMMETRIC DATA)



CENTER - MEAN

SPREAD - STD DEVIATION

(AVG DISTANCE TO MEAN)

• NORMAL DISTRIBUTION MODEL

↳ (APPROXIMATION)

→ THIS MODEL IS USED TO APPROXIMATE PROBABILITIES

→ USED TO COMPARE GROUPS

→ AND MORE...

TWO THINGS THAT DEFINE A "NORMAL DISTRIBUTION"

- ALL HAVE A ① MEAN & A ② STANDARD DEVIATION

↳ μ (MU) & σ (SIGMA)

STATISTICS
SAMPLE

POPULATION

MEAN

\bar{x}

μ (MU)

STD DEV

S

σ (SIGMA)

SIZE

n

N

★ THIS BE WELL ON A QUIZ!

⇒ STATISTICS: #'S REPRESENTING A SAMPLE

⇒ PARAMETER: #'S REPRESENTING A POPULATION

SAMPLE | POPULATION

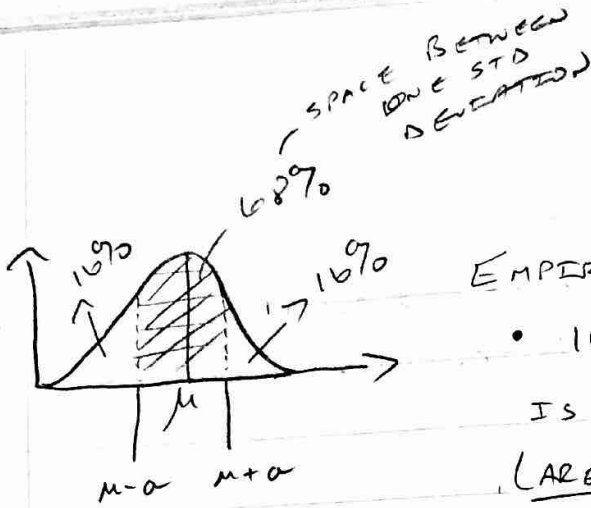
PROPORTION

\hat{p}

p

3-11-19

ONE STANDARD DEVIATION

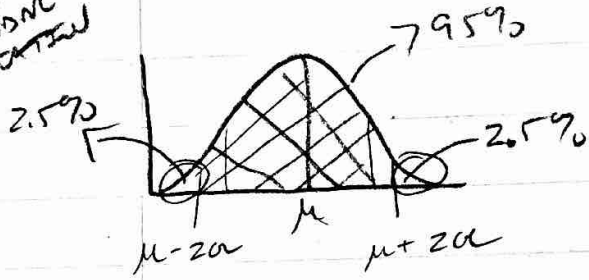


EMPIRICAL RULE: (68-95-99.7)

- 100% OF THE DATA IS BELOW THE CURVE (AREA BELOW CURVE = 1)
- AREA UNDER CURVE = PROBABILITY

- NORMAL DISTRIBUTION DEALS IN CONTINUOUS RANDOM VARIABLE
 ⇒ CAN NOT LIST ALL POSSIBLE VALUES

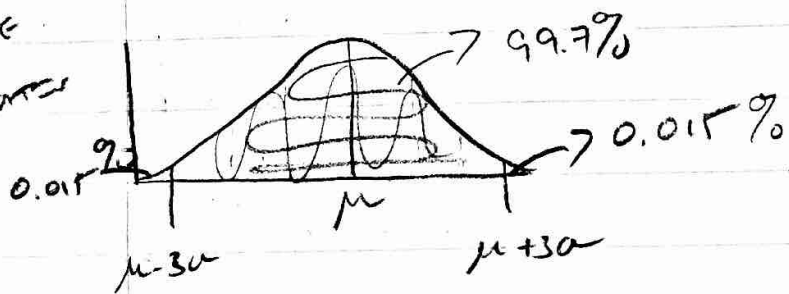
TWO STANDARD DEVIATIONS



- TWO σ'S FROM THE μ CONTAINS WITHIN THE BOUNDARIES, 95% OF THE DATA

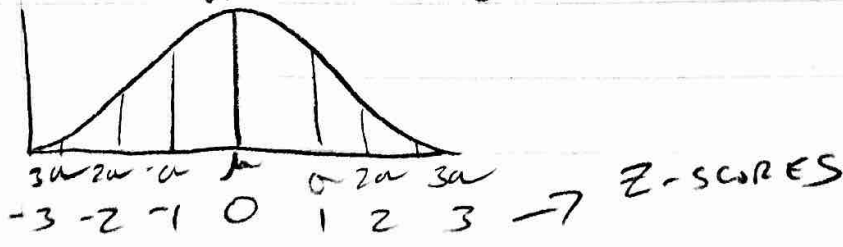
- 2.5% ON EACH WING

THREE STD DEVIATIONS



Z-SCORES

- COUNT THE # OF STANDARD DEVIATIONS FROM THE MEAN



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STANDARD NORMAL (DISTRIBUTION OF Z-SCORES)

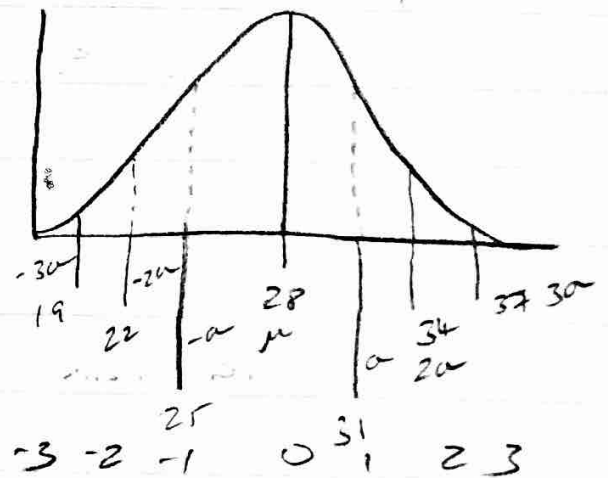
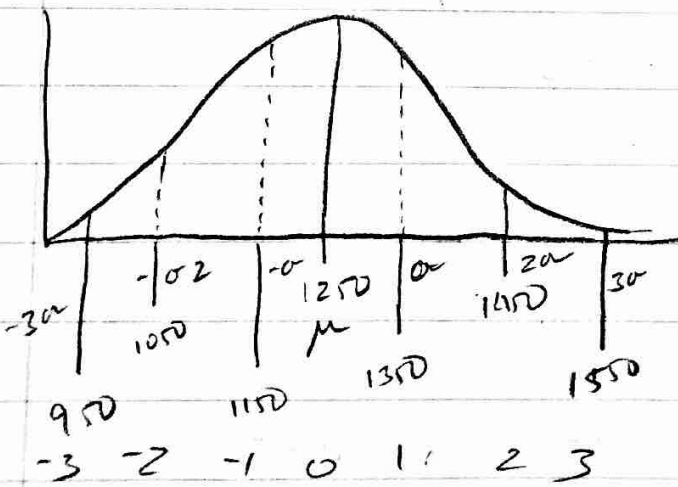
• $\mu = 0$; $\sigma = 1$ For ANY DISTRIBUTION

- $P(-1 < Z < 1) = 68\%$
- $P(Z < -1) = 16\%$
- $P(Z > 2) = 2.5\%$
- $P(-1 < Z < 0) = 34\%$
- $P(Z > -3) = 99.85\%$
- $P(-2 < Z < 1) = 81.5\%$

EXAMPLE: SAT SCORES vs. ACT SCORES

SAT
 $\mu = 1250$
 $\sigma = 100$
 (OUT OF 1600)

ACT
 $\mu = 28$
 $\sigma = 3$
 (OUT OF 36)



1) STUDENT #1 SCORED 1350 ON SAT } WHO DID
 STUDENT #2 SCORED 31 ON ACT } BETTER?

* BOTH PERFORMED SAME: HOW? Z-SCORES MATCH
 1 STD DEV. ABOVE MEANS; PERCENTILES MATCH!

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- 1) CONT: ① Z-SCORES MATCH! BOTH ONE STANDARD DEVIATION ABOVE MEAN
② PERCENTILES MATCH! 84TH PERCENTILE

2) WHICH IS BETTER?

$$\frac{1200 \text{ ON SAT}}{1200 - 1250} = -0.5 \quad \text{OR} \quad \frac{27 \text{ ON SAT}}{27 - 28} = -0.33$$

27

3) WHAT SCORE ON SAT CORRESPONDS TO 22 ON ACT?

$$1050 \text{ SAT} = 22 \text{ ACT}$$

4) WHAT PERCENTILE IS A 1470 ON SAT?
97.5 PERCENTILE

5) WHAT % OF PEOPLE SCORED BELOW 1150 ON SAT? 16%

6) WHAT % OF PEOPLE SCORED BETWEEN 19 AND 31 ON ACT?

$$68 + 16 - 0.15 = 83.85\%$$

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CALCULATING Z-SCORE

$$Z = \frac{(X - \mu)}{\sigma} = \frac{\text{VALUE} - \text{MEAN}}{\text{STDS DEV.}}$$

- TO CALCULATE; SUBTRACT THE KNOWN VALUE X FROM THE MEAN (μ) AND THEN DIVIDE BY THE STANDARD DEVIATION (σ)

EXAMPLES CONT

7) WHAT SAT SCORE CORRESPONDS WITH AN ACT SCORE OF 35? $\frac{35 - 28}{3} = 7/3 = 2.33$

$$1250 + (2.33)(100) = \boxed{1483}$$

8) WHAT PERCENTILE IS THAT SCORE AT?

$\boxed{99.01\%}$ (USED OLI APPLET)
OR WEB CALCULATOR

9) WHAT SCORE IS AT THE 95TH PERCENTILE?

$Z = 1.65$ (FROM Z-CALCULATOR)

$$1250 + (1.65)(100) = 1415 \text{ SAT}$$
$$28 + (1.65)(3) = 32.95 \approx 33 \text{ ACT}$$