AMATYC Contest (Fall 2012) SOLUTIONS

1. [B] If 2 white keys were added, black keys would make up 40% of 90, i.e. 36.
2. [C] . So the height is , i.e. .
3. [C] .
4. [A] Anh, Ana, Ann together is . Replacing Anh's age by Ana's age gives . So .
5. [D] .
6. [E] Since , is at most 4. The prime factorization of for are: , , , . Keep only those cases where all primes of the form appear an even number of times (including 0 times). So only is kept. Any prime of the form (e.g. 29 and 61) can be written as a sum of squares. So . If this is equal to , there are essentially two possibilities for (up to complex conjugation and multiplication by or ), namely or . But we are told that is a prime, so it must be the latter case. Thus . (Remark: The solution relies on knowledge in Gaussian integers.)
7. [C]
8. [B] Each minute they travel a total of mi (making 60 mph.) If Bob left 15 minutes later, there would be minutes when both were moving, cover 51 mi. So Roy moved 9 mi in the 15 min when only he was moving. So Roy moves at 36 mph, and Bob mph.
9. [D] Compare with , namely with .
10. [B] Since 11 is a prime, is a field -- So numbers not divisible by 11 have a reciprocal modulo 11. Now, (mod 11). So we write (mod 11) because the reciprocal of 3 is 4 (, mod 11). So, modulo 11, we have , , , , , , . But is a 1-digit number, so .
11. [E] Need to make sure that is meaningful and that is meaningful and nonzero.
12. [D] . So reads . Since , we have , or the other way around. Therefore , and so .
13. [C] Want , i.e. , i.e. , so. . So or . Namely, , or , or . Since , the answer is .
14. [B] . So it's plausible that . Indeed,

.

1. [A]
2. [D] . To make this an integer, we have for some integer . Thus . The only possibilities for are . So .
3. [E] From we see that is even and is odd. So , are both odd, and , , with , i.e. , therefore . The value gives an integer value .
4. [E] , and . Solve to get .
5. [B] The overlapping area consists of two -- right triangles with the longer leg being 2. So the combined area is .
6. [A] Let . Then are nonnegative integers with and .

Case 1: : .

Case 2: : .

Case 3: : .

Case 4: : .

So the answer is .