

## Intermediate Algebra is a prerequisite (a must!)

What does knowing Intermediate Algebra mean? It means you already know the following...

**1. How to work with exponents** (we don't use logarithms in 1A, but you'll need them for 1B). This will help you gauge if a particular calculation you perform makes sense (*We will not rely solely on our calculators!*).

$$a^0 = 1 \quad ; \quad a^1 = a \quad ; \quad 1^n = 1 \quad ; \quad a^{-n} = 1/a^n \quad [e.g. \quad 2^{-3} = 1/2^3 = 0.125 \quad ]$$

$$a^n \cdot a^m = a^{n+m} \quad [e.g. \quad 2^3 \cdot 2^4 = 2^7 = 128 \quad ] \quad ; \quad a^n \cdot b^n = (a \cdot b)^n \quad [e.g. \quad 3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144 \quad ]$$

$$\frac{a^n}{a^m} = a^{n-m} \quad [e.g. \quad \frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4 \quad ] \quad ; \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad [e.g. \quad 4^3/2^3 = \left(\frac{4}{2}\right)^3 = 8 \quad ]$$

$$(b^n)^m = b^{n \cdot m} \quad [e.g. \quad (2^3)^2 = 2^6 = 64 \quad ] \quad ; \quad b^{n/m} = \sqrt[m]{b^n} \quad [e.g. \quad 2^{\frac{6}{2}} = \sqrt[2]{2^6} = \sqrt{(2^3)^2} = 2^3 = 8 \quad ]$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad [e.g. \quad \sqrt[3]{128} = \sqrt[3]{64 \cdot 2} = \sqrt[3]{64} \cdot \sqrt[3]{2} = 4\sqrt[3]{2} \quad ] \quad ; \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad [e.g. \quad \sqrt[3]{\frac{1}{64}} = \frac{\sqrt[3]{1}}{\sqrt[3]{64}} = \frac{1}{4} \quad ]$$

Do the following problems (no calculator!), write your answer with just one digit before the decimal and a power of ten (this is what we call scientific notation and we'll use it extensively.).

$$a. \quad \frac{(3 \times 10^{23})(8 \times 10^{41})}{2 \times 10^{-45}} = \quad ; \quad b. \quad (3.2 \times 10^{105}) + \frac{(4.6 \times 10^{148})}{(2.0 \times 10^{44})} =$$

(Answer: a.  $1.2 \times 10^{110}$  ; b.  $3.43 \times 10^{105}$ )

### 2) How to solve systems of linear equations with many variables.

Example:

$$\begin{aligned} x + z &= 6 \\ z - 3y &= 7 \\ 2x + y + 3z &= 15 \end{aligned}$$

(Answer:  $x = 2$  ;  $y = -1$  ;  $z = 4$ )

**3) Manipulate equations to solve them for any given variable.** Check the examples below; all of them are taken from equations we will actually work with, so, you must be able to follow the steps (Note: in algebra we may arrive to the same result in different ways... i.e. you may use a different technique):

**a.** This equation will be obtained in Chapter 10 (Gases). It will help you solve ALL problems (Chem 1A type) that involve a change in the conditions of a gas. The subscripts indicate different values (initial and final).

$$\frac{V_i P_i}{n_i R T_i} = \frac{V_f P_f}{n_f R T_f} \quad \text{Solve for } T_f \quad ; \quad \frac{V_i P_i \cancel{n_f R}}{\cancel{n_i R} T_i V_f P_f} = \frac{1}{T_f} \quad ; \quad T_f = \frac{n_i T_i V_f P_f}{V_i P_i n_f}$$

Notice how  $R$  cancels out since it is the only constant (no subscript).

**b.** This Equation is introduced on Chapter 6 (Electronic Structure of Atoms). The terms  $n_f$  and  $n_i$  refer to a final and initial value (respectively); whereas  $\Delta E$  indicates a change in energy:

$$\Delta E = -2.18 \times 10^{-18} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad \text{solve for } n_f \quad ; \quad \frac{\Delta E}{-2.18 \times 10^{-18}} = \frac{1}{n_f^2} - \frac{1}{n_i^2} \quad ;$$

$$\frac{1}{n_f^2} = \frac{\Delta E}{-2.18 \times 10^{-18}} + \frac{1}{n_i^2} \quad ; \quad n_f^2 = \frac{-2.18 \times 10^{-18}}{\Delta E} + n_i^2 \quad ; \quad n_f = \sqrt{\frac{-2.18 \times 10^{-18}}{\Delta E} + n_i^2}$$

**c.** The following equation is what we call Graham's Law (also related to gases).

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{3RT}{M_1}}}{\sqrt{\frac{3RT}{M_2}}} \quad \text{Solve for } M_1 \quad ; \quad \frac{v_1}{v_2} = \left( \frac{\frac{3RT}{M_1}}{\frac{3RT}{M_2}} \right)^{1/2} \quad ; \quad \left( \frac{v_1}{v_2} \right)^2 = \frac{3RT M_2}{3RT M_1} = \frac{M_2}{M_1}$$

$$\frac{1}{M_1} = \frac{\left( \frac{v_1}{v_2} \right)^2}{M_2} \quad ; \quad M_1 = \frac{M_2}{\left( \frac{v_1}{v_2} \right)^2}$$

**d.** The equation below will be obtained on Chapter 5 (Thermochemistry); it allows you to find – among other things – the final temperature ( $T_f$ ) that will be reached when you put in contact two substances with different temperatures.

$$m_1 C_1 (T_f - T_1) = -m_2 C_2 (T_f - T_2) \quad \text{Solve for } T_f$$

$$m_1 C_1 T_f - m_1 C_1 T_1 = -m_2 C_2 T_f + m_2 C_2 T_2 \quad ; \quad m_1 C_1 T_f + m_2 C_2 T_f = m_1 C_1 T_1 + m_2 C_2 T_2$$

$$T_f (m_1 C_1 + m_2 C_2) = m_1 C_1 T_1 + m_2 C_2 T_2 \quad ; \quad T_f = \frac{m_1 C_1 T_1 + m_2 C_2 T_2}{(m_1 C_1 + m_2 C_2)}$$

**e.** The final example is an equation obtained (Chapter 1) to solve a problem in which two metals (Copper and Zinc) are combined to obtain a new substance (an alloy called Brass).

$$\frac{m_{\text{Copper}}}{d_{\text{Copper}}} + \frac{(m_{\text{Brass}} - m_{\text{Copper}})}{d_{\text{Zinc}}} = V_{\text{Brass}} \quad \text{Solve for } m_{\text{Brass}}$$

$$\frac{(m_{\text{Brass}} - m_{\text{Copper}})}{d_{\text{Zinc}}} = V_{\text{Brass}} - \frac{m_{\text{Copper}}}{d_{\text{Copper}}} \quad ; \quad m_{\text{Brass}} - m_{\text{Copper}} = \left( V_{\text{Brass}} - \frac{m_{\text{Copper}}}{d_{\text{Copper}}} \right) d_{\text{Zinc}}$$

$$m_{\text{Brass}} = \left( V_{\text{Brass}} - \frac{m_{\text{Copper}}}{d_{\text{Copper}}} \right) d_{\text{Zinc}} + m_{\text{Copper}}$$

#### 4) How to come up with a mathematical model (equation) from a word problem.

This is, perhaps, the core idea behind anyone studying mathematics (algebra in this case): To be able to translate a problem from our spoken/written language to the language of mathematics (to get an equation or equations); then, solve the problem in the realm of mathematics; and finally, translate the answer back to our common language (*i.e.* assess what the answer means and evaluate if it makes sense – this is why point 1 above matters.)

To master this skill, you must be able to identify all relationships (explicit and implicit) in the problem.

##### Example 1:

a. By which factor will the volume of a cylinder increase if we double its radius? ( $V_{\text{cylinder}} = \pi r^2 h$ )

b. If the initial volume is 5 gallons (gal), what will the final volume be?

*Answer: A "factor" means a ratio (a comparison of how many times larger or smaller something is in relation to a standard or unit object/concept of a known/given size); therefore, the equation needed here is:*

*We want the "factor" or ratio:  $\frac{V_2}{V_1} = ?$  ;  $V_2$  being the final volume after the change.*

*Since the equation to calculate the volume of a cylinder is given explicitly, we can write:*

*Initial and final volumes of the cylinder:  $V_1 = \pi r_1^2 h_1$  and  $V_2 = \pi r_2^2 h_2$ . So:  $\frac{V_2}{V_1} = \frac{\pi r_2^2 h_2}{\pi r_1^2 h_1}$*

*The following relations are also given:*

*The radius is doubled:  $r_2 = 2r_1$  (explicitly given) ;  $h_2 = h_1 = h$  (unchanged – implicitly given)*

*The relationships obtained above (the system of three equations) can be used to solve the problem as follows:*

$$\text{a. } \frac{V_2}{V_1} = \frac{\pi r_2^2 h_2}{\pi r_1^2 h_1} = \frac{\pi (2r_1)^2 (h)}{\pi r_1^2 h} = 2^2 = 4 \text{ times larger} ; \quad \text{b. } V_2 = 4 \times V_1 = 4(5 \text{ gal}) = 20 \text{ gal}$$

##### Example 2:

Using the equation for the gases obtained above:

$$T_f = \frac{n_i T_i V_f P_f}{V_i P_i n_f}$$

Suppose you make all final values of  $n$ ,  $V$  and  $P$  three times larger than the initial ones. How much larger or smaller will  $T_f$  be compared to  $T_i$ .

*Since all values are three times larger than the initial ones, we get:  $n_f = 3n_i$  ;  $V_f = 3V_i$  and  $P_f = 3P_i$ .*

*Replacing these three relationships on the equation given, so as to obtain an equation in terms of the initial values, we get:*

$$\frac{T_f}{T_i} = \frac{n_i V_f P_f}{V_i P_i n_f} = \frac{n_i (3V_i) (3P_i)}{V_i P_i (3n_i)} = \frac{9}{3} = 3 \quad (T_f \text{ will be 3 times larger than } T_i)$$

**Final Note:** The equations used on this handout will not make much sense to you; that is, until we cover the theory behind them. Nevertheless, the algebra must be clear... if it is not, strengthen your algebra and then take Chem 1A the next term. You will not have the time to strengthen your algebra and learn chemistry at the same time; that is why it is a prerequisite. Please be conscious of this fact.