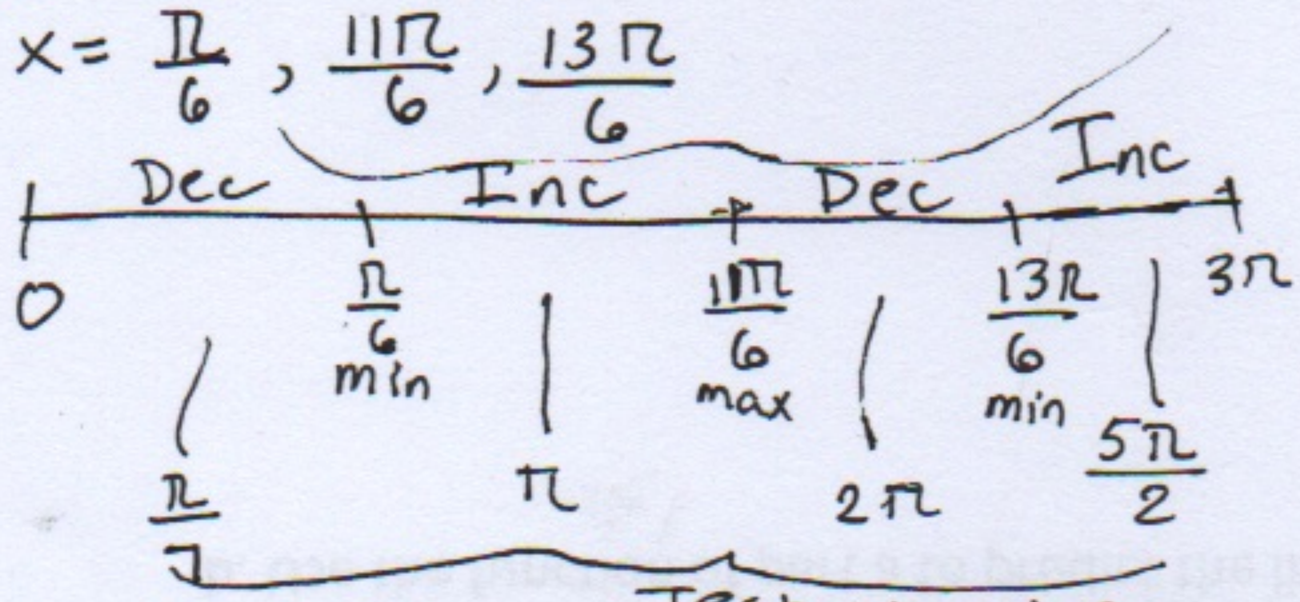
Math 3A Solutions Test 3 Spring 2018 p. 2

6 a. $y' = \frac{\sqrt{3}}{2} - \cos x = 0$

$\cos x = \frac{\sqrt{3}}{2}$

Critical numbers:

$x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$



$\frac{\sqrt{3}}{2} - \cos\left(\frac{\pi}{7}\right) = -$ Dec

$\frac{\sqrt{3}}{2} - \cos \pi = +$ Inc

$\frac{\sqrt{3}}{2} - \cos 2\pi = -$ Dec

$\frac{\sqrt{3}}{2} - \cos \frac{5\pi}{2} = +$ Inc

$\boxed{\begin{array}{l}\text{Increasing: } \left(\frac{\pi}{6}, \frac{11\pi}{6}\right), \left(\frac{13\pi}{6}, 3\pi\right) \\ \text{Decreasing: } \left(0, \frac{\pi}{6}\right), \left(\frac{11\pi}{6}, \frac{13\pi}{6}\right)\end{array}}$

6b. $f\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}\left(\frac{11\pi}{6}\right) - \sin\left(\frac{11\pi}{6}\right)$

$= \frac{11\pi\sqrt{3}}{12} + \frac{1 \cdot 6}{2 \cdot 6}$

$= \frac{11\pi\sqrt{3} + 6}{12}$

$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \cdot \frac{\pi}{6} - \sin \frac{\pi}{6}$

$= \frac{\pi\sqrt{3}}{12} - \frac{1 \cdot 6}{2 \cdot 6}$

$= \frac{\pi\sqrt{3} - 6}{12}$

$f\left(\frac{13\pi}{6}\right) = \frac{\sqrt{3}}{2} \cdot \frac{13\pi}{6} - \sin\left(\frac{13\pi}{6}\right)$

$= \frac{13\pi\sqrt{3}}{12} - \frac{1 \cdot 6}{2 \cdot 6}$

$= \frac{13\pi\sqrt{3} - 6}{12}$

$\boxed{\begin{array}{l}\text{Local Max: } \left(\frac{11\pi}{6}, \frac{11\pi\sqrt{3}+6}{12}\right) \\ \text{Local Min: } \left(\frac{\pi}{6}, \frac{\pi\sqrt{3}-6}{12}\right) \\ \quad\text{and} \\ \left(\frac{13\pi}{6}, \frac{13\pi\sqrt{3}-6}{12}\right)\end{array}}$

$\left(\frac{11\pi}{6}, 5.49\right)$

$\left(\frac{\pi}{6}, -.047\right)$

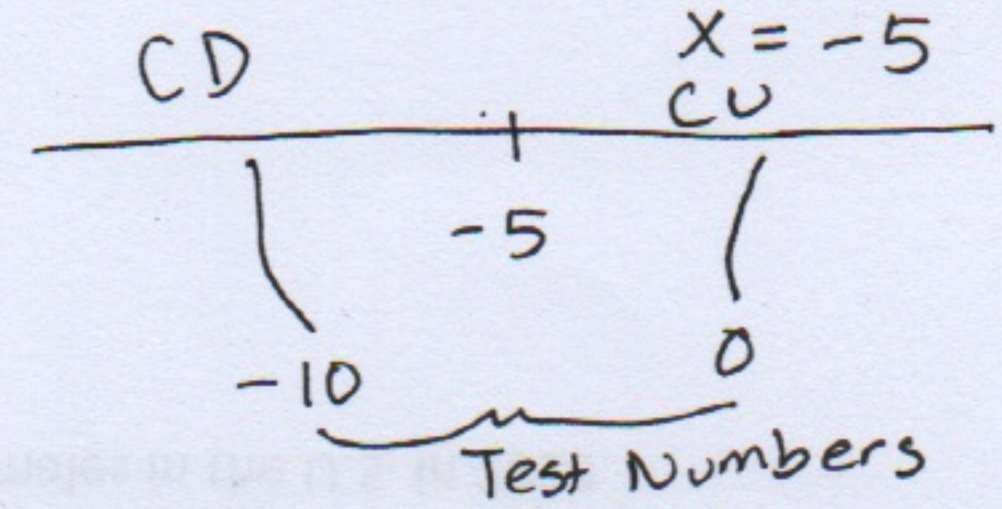
$\left(\frac{13\pi}{6}, 5.39\right)$

7 a. $f'(x) = 27x^2 + 270x - 11$

$f''(x) = 54x + 270 = 0$

$\frac{54x}{54} = \frac{-270}{54}$

$x = -5$



$f''(-10) = 54(-10) + 270 < 0$ CD

$f''(0) = 54(0) + 270 > 0$ CU

$\boxed{\begin{array}{l}\text{Concave Up } (-5, \infty) \\ \text{Concave Down } (-\infty, -5)\end{array}}$

7b. $f(-5) = 9(-5)^3 + 135(-5)^2 - 11(-5)$

$= 2305$

$\boxed{\text{Inflection Point: } (-5, 2305)}$

8. $\frac{f(b) - f(a)}{b - a} = \frac{\ln 7 - \ln 1}{7 - 1} = \frac{\ln 7}{6}$

$f'(x) = \frac{1}{x}$

$\frac{1}{x} = \frac{\ln 7}{6}$

$\frac{x \ln 7}{\ln 7} = \frac{6}{\ln 7}$

$x = \frac{6}{\ln 7}$

$\boxed{c = \frac{6}{\ln 7}}$

9a. $\lim_{x \to 2} \frac{3x^2 - 10x}{3x^2} = \frac{3(2)^2 - 10(2)}{3(2)^2} = \frac{-8}{12} = \boxed{-\frac{2}{3}}$

9b. $\lim_{x \to \infty} \frac{9\sin\left(\frac{\pi}{x}\right)}{\frac{1}{x}} = \lim_{x \to \infty} \frac{9\cos\left(\frac{\pi}{x}\right) \cdot \left(-\frac{\pi}{x^2}\right)}{-\frac{1}{x^2}}$

$= \lim_{x \to \infty} 9\cos\left(\frac{\pi}{x}\right) \cdot \pi$

$= 9\cos 0 \cdot \pi$

$= \boxed{9\pi}$